Stacked Revenues Maximization of Distributed Battery Storage Units Via Emerging Flexibility Markets

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***Abstract*—Power system flexibility is seen as a key for addressing the management challenges in the future smart grids. Distributed Energy Resources (DERs) can provide the necessary flexibility to: a) maintain a stable frequency and a secure energy supply in an overall system perspective, and b) maintain bus voltages and secure transfer capacities in their local networks. In this context, Flexibility Service Providers (FSPs) facilitate the management of the transmission and the distribution network, while at the same time they optimally exploit their DERs. This work proposes a bilevel model for an FSP owning distributed Battery Storage Units and participating in: i) wholesale Energy, Reserve and Balancing Markets, and ii) a novel distribution-level Flexibility Market. The developed model is applied to the IEEE 33-bus radial distribution system and the results demonstrate that it achieves superlinear gains. Finally, a sensitivity analysis is conducted to study the impact of several externalities on the FSP's decisions.**

***Index Terms*—Flexibility Markets, Battery Storage Units, Stacked Revenues, Bilevel Model.**

Acronyms

BM	Balancing Market
BSU	Battery Storage Unit
DAM	Day-Ahead Electricity Market
DER	Distributed Energy Resource
DG	Distributed Generator
DLFM	Distribution-level Flexibility Market
DN	Distribution Network
DSO	Distribution System Operator
FMO	Flexibility Market Operator
FSP	Flexibility Service Provider
MILP	Mixed Integer Linear Program
OPF	Optimal Power Flow
RM	Reserve Market
TN	Transmission Network
TSO	Transmission System Operator

Nomenclature

A. Sets

$X^{\text{UL / RM / FM}}$	Sets of optimization variables of the upper- and lower-level problems
H	Set of timeslots in the scheduling horizon
S	Set of BSUs
G	Set of generators participating in the reserve market
N	Set of nodes of the distribution network
B	Set of branches of the distribution network
F_r	Set of competing FSPs
$\Omega_d(n)/\Omega_p(n)$	Set of decedent/precedent nodes connected to node n of the distribution network
Ω	Set of scenarios concerning the balancing market prices

B. Subscripts and Superscripts

t	Subscript indicating the timeslot
i	Subscript indicating the resources
n,j,k	Subscript indicating the nodes
nk	Subscript indicating the network lines connecting nodes n and k
ω	Subscript indicating the balancing market price scenarios
s	Superscript for the BSUs
g	Superscript for the generators in the reserve market
r	Superscript for the competitors in the flexibility market

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P/Q	Superscript for the active/reactive power in the distribution network
up/dn	Superscript for the upward/downward services
e/b	Superscript for the energy/balancing market
v	Iteration counter of the proposed algorithm

C. Variables

$\widehat{[\cdot]}$	Indicates quantity offers
$\underline{[\cdot]}/\overline{[\cdot]}$	Indicates minimum/maximum bounds
dis / ch	Scheduled BSU's discharge/charge power sold/bought in the wholesale energy market
r	Reserve capacity commitment
p/q	Flexibility market active/reactive power dispatch
$p^{\mathrm{BSU}}/q^{\mathrm{BSU}}$	BSUs' overall active/reactive power schedules
h	Binary variable indicating the operating mode of BSUs
E	State of energy of BSUs
c	Price bid of the FSP
f	Power flow in distribution network
U	Square voltage magnitude
λ	Market prices
ϕ, ψ	Dual variables of the reserve and flexibility markets clearing processes

D. Parameters

T	Last timeslot of the scheduling horizon
$\overline{S}$	Apparent power rating of converter of BSU
$\eta^{\mathrm{c}}/\eta^{\mathrm{d}}$	Charge / Discharge efficiency
$\tilde{c}$	Price bids of the competitors
R	System's reserve capacity requirement
d / g	Scheduled demand / generation in the distribution network
$\delta^{\mathrm{d}}/\delta^{\mathrm{g}}$	Parameters converting active power into their reactive power – tan(arccos(power factor))
r / x	Resistance / Reactance of branches
ξ_ω	Probability of scenario ω
ϵ	Convergence tolerance of the proposed algorithm

I. Introduction

A. Motivation and Aim

THE ongoing decarbonisation and decentralization of the electric power landscape delivers clean, sustainable and low-cost energy as well as energy autonomous societies [1]. On the other hand, the rapid proliferation of distributed, variable and unpredictable generation can result in various challenges for the network operators, such as line and transformer congestion, voltage limit violations, and eventually dramatically increase the demand for flexibility [2]. Using the power system's flexibility instead of costly network investments can create financial opportunities for the end users facilitating the integration of renewable energy resources. Thus, Distributed Energy Resources (DERs) can provide the necessary flexibility services at both the distribution and the transmission level, as long as an economically efficient market environment is designed to motivate the investments in such technologies [3].

In today's power sector, the procurement of flexibility is characterized by a monopsony, since the Transmission System Operator (TSO) is the main buyer of such services. In addition, the interaction between the TSO and the Distribution System Operators (DSOs) is insufficient and the clearing process of the wholesale energy markets does not take into account the distribution grid operation. Consequently, the participation of distributed generators (DGs) and other DERs in such markets can lead to violations of the physical constraints that the Distribution Network (DN) imposes and, consequently, inefficient (technically and economically) market results. The latter dictates the need for a shift of the DSO's role towards a more active network operator, which will be able to purchase flexibility services from the local DERs.

In the context of EC-funded H2020 FLEXGRID project [4], the aforementioned issues are addressed by the development of a distribution level Flexibility Market (DLFM). In more detail, Flexibility Service Providers (FSPs), e.g. energy storage owners, demand response aggregators, DG owners, capacitor banks, etc., declare their flexibility capacity and cost to a Flexibility Market Operator (FMO), which in turn clears the DLFM by minimizing the cost of acquiring the flexibility needed to ensure the participation of the DERs in the wholesale markets without jeopardizing the operation of the distribution grid. The ultimate goal of FLEXGRID is to test, evaluate and compare various DLFM architectures (or else to quantify the impact of DLFM positioning in the current EU regulatory framework). In this paper, we focus on the reactive DLFM architecture (i.e. DLFM reacts to the market clearing decisions made by the transmission-level markets), as it is compatible with the existing regulation [5].

In this market environment, a merchant owner of Battery Storage Units (BSUs) can increase its profitability by providing energy and ancillary services at both the transmission and the distribution level. BSUs with smart AC/DC converters can provide valuable grid services to the TSOs and DSOs [6], such as peak shaving, energy (wholesale energy and regulation) and power (frequency containment) balancing, alleviation of grid contingencies (voltage and congestion issues), black-start services, etc. In this work we consider an FSP that owns a set of distributed BSUs and provides services to both the system-wide grid (TSO) and the local distribution network (DSO).

There is a great deal of studies that have dealt with the problem of optimizing the multi-service portfolio of merchant-owned BSUs. Works in [7] and [8] studied the optimal bidding of a BSU in the day-ahead and real-time energy-only markets, while [9] and [10] dealt with energy storage devices participating in energy and frequency regulation markets. Authors in [11] and [12] studied the problem of optimal bidding and operating strategies for a storage owner participating in the energy and performance-based regulation markets. Similarly, [13] and [14] considered storage units participating in the day-ahead energy and reserve, as well as the real-time energy and regulation markets. While the aforementioned works considered storage units that cannot affect the market prices and acting only as price takers, works in [15] and [16] used bilevel programming to model the revenue maximization problem of a merchant storage owner acting as a price maker in transmission-level energy and reserve markets. All these works differ from our study as they optimize the participation of storage units in only transmission-level energy and ancillary services' markets.

Another strand of research considered distributed BSUs that provide services to both the transmission and distribution systems. Authors in [17] consider a storage owner simultaneously participating in three markets: energy, TSO ancillary services and DSO (congestion) market. The authors proposed a portfolio theory-based approach to decide the optimal storage capacity allocated to each market in order to maximize the benefits at minimum risk. The DSO services' remuneration is based on the congestion cost savings and is calculated based on a congestion cost index. Work in [18] formulated a Mixed-Integer Linear Program (MILP) to model the profit maximization problem of a storage that provides system-wide (energy arbitrage and system balancing) and local network services (peak demand shaving to alleviate the distribution network congestion). The DSO services' remuneration is assumed to be equal to the opportunity cost of a storage plant associated with the DSO's services, i.e. its revenue increase from the energy and balancing markets when no storage capacity is allocated to provide the DSO services. Work in [19] maximized the aggregated profits of an energy storage providing energy, reserve and frequency regulation services to the transmission system and congestion management to the distribution grid. The distribution grid services are considered compulsory and are not remunerated. A model predictive control approach was employed in [20] to dynamically allocate storage power and energy capacities to either a local or a grid service with the objective of maximizing the profit of an energy storage aggregator. The energy storage profits result from energy price arbitrage and primary frequency control minus the costs of load curtailment reduction and transformer overheating. In [21], a generic formulation of the scheduling problem of a multi-service energy storage owner was designed. Based on this generic framework, the authors decide on the portion of energy and power to be allocated for dispatching the operation of a medium-voltage feeder and providing primary frequency control services. Moreover, the authors in [22] proposed a joint optimization framework for energy storage units to reduce energy bills of commercial consumers (peak shaving) and seek profit through the provision of frequency regulation services. Unlike these works, we consider a distribution-level marketplace, which determines the magnitude of the local grid services and their compensation through solving an Optimal Power Flow (OPF) problem.

Lastly, the bilevel interdependencies between two markets result in bilinear terms in the objective function which cannot be solved using the standard linearization techniques (big-M [23] and exact linearization methods [24]). Works in [15] and [25] use the binary expansion method [26] to deal with this source of non-linearity. However, this approach increases complexity by adding new binary variables. In contrast to [15] and [25], we adopt a novel iterative approach that avoids the extra computational burden of the binary expansion method.

B. Paper Positioning and Contribution

In light of the recent smart grid architectural progress in the development of distribution-level flexibility markets [27], this work co-optimizes the transmission and distribution grid services provided by an FSP owning distributed BSUs as in [17]–[22], using bilevel programming as in [15] and [16]. By considering market scales, we assume that the FSP is acting as a price maker in the Reserve Market (RM) and the DLFM, while it cannot affect the market prices in the wholesale energy and balancing markets. Thus, the contribution of this work lies in the following:

1) It proposes a novel energy market architecture, in which a DLFM is introduced in the timeframe between the day-ahead energy and the balancing markets. An innovative DLFM clearing process is proposed, which enables the DSO to buy the needed flexibility to tackle the possible contingencies resulting from the wholesale energy market dispatch decision, calculating the optimal flexibility dispatch and compensation.
2) A new bidding strategy is proposed for an FSP that stacks revenues based on four products: 1) wholesale energy arbitrage, 2) reserve capacity and 3) balancing energy for the TSO, and 4) local constraint support for the DSO. Bilevel modeling is used to model the strategic participation of a BSUs' owner in both the TSO and DSO markets.
3) A novel iterative process is proposed to deal with non-linearities due to the FSP's participation in two interdependent markets.

To the best of our knowledge, this is the first work that uses Bilevel programming to model the decision process of a strategic FSP owning distributed BSUs and providing services both system-wide and to the local network operator. This paper's structure is organized as follows: Section II describes the proposed market architecture and the bilevel structure of the problem under discussion. Section III presents the solution method. Section IV provides a detailed evaluation of the proposed solution. Finally, Section V concludes the paper and discusses future work.

II. System Model and Problem Formulation

This work presents a market architecture in which a DLFM follows in an optimal way the decisions made by the DN-unaware day-ahead energy and reserve markets (intra-day timeframe), without changing the existing TSO wholesale market structure being thus compatible with the existing regulatory framework (Fig. 1). This Reactive DLFM (R-DLFM) architecture enables: a) the DERs to participate in the TSO wholesale markets without jeopardizing the smooth operation of their underlying network, and b) the DSO to buy the needed flexibility to remove contingencies resulting from the wholesale energy market dispatch process.

In a first step, as shown in Fig. 1, the Market Operator (MO) runs the Transmission Network (TN)-level day-ahead energy market after the TN-level Energy Service Providers (ESPs), such as generating companies, demand aggregators, retailers, etc., and the DN-level FSPs having submitted their energy offers/bids. Subsequently, the TSO operates the day-ahead reserve market given the MO's dispatch schedules (DAM dispatch) and the reserve capacity offers from the RM participants. This practice is common in most European markets (e.g. Nord Pool,

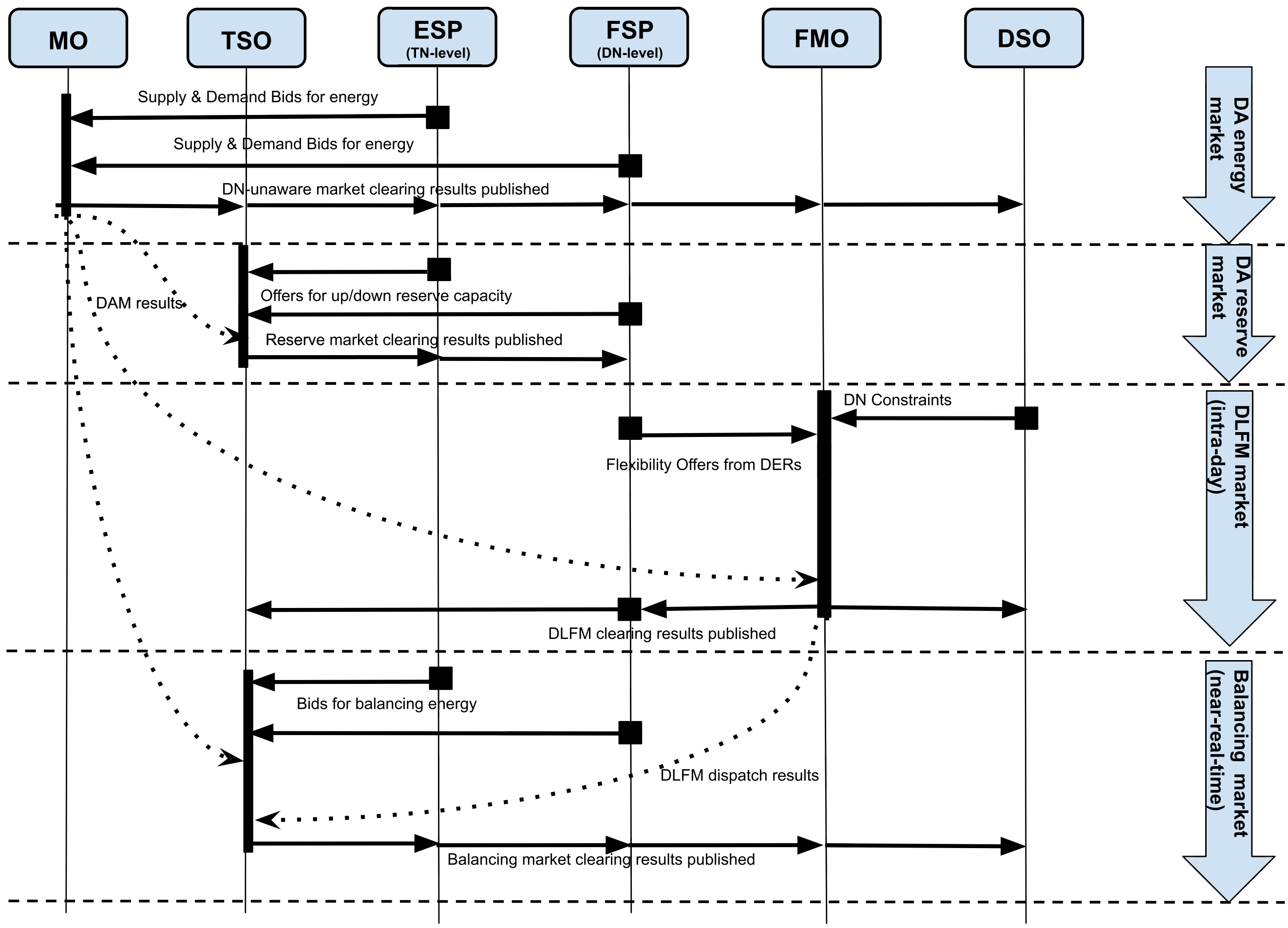

Fig. 1. Proposed Reactive Distribution-Level Flexibility Market (R-DLFM) architecture.

EPEX, OMEL, GME, MIBEL), where the energy and reserve markets are sequentially cleared ([28], [29]). The role of the RM is to provide to the TSO the required upward/downward reserve capacity to keep its system balanced in the real-time (balancing) stage. In the third step, the distribution-level FSPs submit their flexibility offers (active and reactive up/down flexibility) to the FMO, which in turn clears the local DLFM, taking into consideration the DAM results, the particularities and the constraints of the DN (provided by the DSO), thus performing the DN-aware market clearing. The role of the DLFM is to ensure that the DN operates within its safety limits, i.e. to remove local congestion, local balancing and voltage control issues that might occur due to the DN-unaware DAM clearing process. Thus, the FMO clears the DLFM by running an OPF problem, which takes as input: i) the MO's decisions pertaining to the local DERs that participate in the DAM, ii) the active/reactive up/down flexibility offers submitted by the FSPs and iii) the DN constraints provided by the DSO. In case the TN-level DAM has not produced dispatches that violate the DN constraints, the DLFM results in zero flexibility procurement and, of course, zero DLFM prices. Otherwise, the DLFM produces non-zero active/reactive and upward/downward flexibility dispatches and the corresponding flexibility prices per DN node at which the FSPs will be paid for their services. Therefore, the DLFM clearing process will re-adjust the DAM position of the DERs located in the specific DN. Thus, these DERs will have to balance their portfolio in the TSO's balancing market (sell/buy power), in order to respect their commitment to the MO (DAM dispatches). For more details regarding the market architecture, we kindly refer an interested reader to [5], [30].

In the context of the proposed R-DLFM architecture, we propose a bidding strategy of a profit-seeking FSP that owns a set of BSUs located at various nodes of a radial DN and participates in the TN-level energy, reserve and balancing markets, as well as in the DLFM. We assume that the FSP cannot affect the DAM and BM prices (acts as a price taker), while its total BSUs' capacity is able to influence the RM and the DLFM prices. The objective of the FSP is to maximize its stacked revenues by optimizing its bidding strategy in the four aforementioned markets. The FSP submits: 1) self-scheduling bids in the DAM and BM, 2) price-quantity pairs for upward and downward reserve capacity in the RM, and 3) price-quantity pairs for four products in the DLFM, i.e. i) upward active power (MW – € /MW), ii) downward active power (MW – € /MW), iii) upward reactive power (MVAr – € /MVAr), and iv) downward reactive power (MVAr – € /MVAr). Uncertainties pertaining

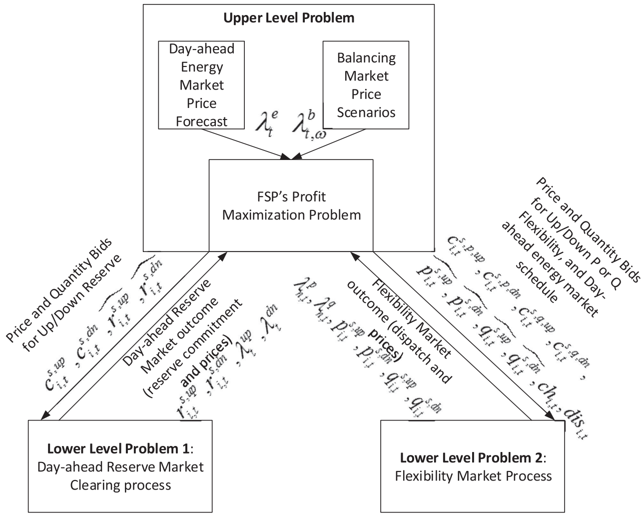

Fig. 2. The proposed bilevel model.

to market competition and local grid consumption/production power are not considered. We perform a deterministic analysis, allowing us to focus on studying the interactions between the individual markets, and how the FSP can manage its BSU portfolio to increase its profitability by participating in the four markets in a co-optimized manner. A stochastic optimization technique can be transparently implemented in the proposed model to tackle the aforementioned uncertainties. In this case, however, an extensive computational burden would be added, so mathematical approaches such as decomposition techniques or robust optimization could offer interesting studies and promising solutions.

A bilevel model (Fig. 2) is proposed to formulate the FSP's problem of determining the optimal bidding strategy and the charging/discharging schedule of the BSUs. In the upper level the FSP decides on the BSUs' operating schedule and its bidding strategy, while taking as input the day-ahead energy prices and balancing market forecast prices and anticipating the impact of its decisions on the reserve and flexibility markets. The FSP's decisions include the energy traded in the day-ahead energy market, the price and quantity bids to the RM and DLFM and the power bought/sold in the BM. In the lower-level, for given FSP's decisions, the TSO and the FMO clear the RM and the DLFM, respectively. In the RM and the DLFM clearing processes the bids of the other market participants are treated as parameters. Also, the decisions of the DAM concerning the distribution-level demand and production are also treated as input parameters in the DLFM clearing process.

A. Upper-Level Problem: Profit Maximization

The upper-level problem maximizes the FSP's profits in various markets by selecting the optimal bidding/offering schedule and is formulated below.

$$\min_{X^{UL}} \sum_{t\in H} \Bigg(\sum_{i\in S} \big(\lambda_t^{\mathrm{e}} \cdot (ch_{i,t} - dis_{i,t}) - \lambda_t^{\mathrm{up}} \cdot r_{i,t}^{\mathrm{s,up}} - \lambda_t^{\mathrm{dn}} \cdot r_{i,t}^{\mathrm{s,dn}}$$

$$- \lambda_{i,t}^{\mathrm{P}} \cdot (p_{i,t}^{\mathrm{s,up}} - p_{i,t}^{\mathrm{s,dn}}) - \lambda_{i,t}^{\mathrm{Q}} \cdot (q_{i,t}^{\mathrm{s,up}} - q_{i,t}^{\mathrm{s,dn}})$$

$$- \sum_{\omega\in\Omega} \xi_\omega \cdot \lambda_{t,\omega}^{\mathrm{b}} \cdot (p_{i,t}^{\mathrm{s,up}} - p_{i,t}^{\mathrm{s,dn}}) \big) \Bigg) \tag{a.1}$$

Subject to

$$0 \le dis_{i,t} \le h_{i,t} \cdot \overline{S_i} \;\; \forall i \in S, t \in H \tag{a.2}$$

$$0 \le ch_{i,t} \le (1 - h_{i,t}) \cdot \overline{S_i} \;\; \forall i \in S, t \in H \tag{a.3}$$

$$h_{i,t} \in \{0, 1\} \;\; \forall i \in S, t \in H \tag{a.4}$$

$$0 \le \widehat{r_{i,t}^{\mathrm{s,up}}} \le \overline{S_i} + (ch_{i,t} - dis_{i,t}) \;\; \forall i \in S, t \in H \tag{a.5}$$

$$0 \le \widehat{r_{i,t}^{\mathrm{s,dn}}} \le \overline{S_i} - (ch_{i,t} - dis_{i,t}) \;\; \forall i \in S, t \in H \tag{a.6}$$

$$0 \le \widehat{p_{i,t}^{\mathrm{s,up}}} \le \overline{S_i} + (ch_{i,t} - dis_{i,t} - \widehat{r_{i,t}^{\mathrm{s,up}}}) \;\; \forall i \in S, t \in H \tag{a.7}$$

$$0 \le \widehat{p_{i,t}^{\mathrm{s,dn}}} \le \overline{S_i} + (dis_{i,t} - ch_{i,t} - \widehat{r_{i,t}^{\mathrm{s,dn}}}) \;\; \forall i \in S, t \in H \tag{a.8}$$

$$E_{i,t} = E_{i,t-1} - (dis_{i,t} + p_{i,t}^{\mathrm{s,up}})/\eta_i^{\mathrm{d}} + \eta_i^{\mathrm{c}} \cdot (ch_{i,t} + p_{i,t}^{\mathrm{s,dn}})$$
$$\forall i \in S, t \in H \tag{a.9}$$

$$E_{i,t} + \widehat{r_{i,t}^{\mathrm{s,dn}}} \cdot \eta_i^{\mathrm{c}} \le \overline{E}_i \;\; \forall i \in S, t \in H \tag{a.10}$$

$$E_{i,t} - \widehat{r_{i,t}^{\mathrm{s,up}}}/\eta_i^{\mathrm{d}} \ge \underline{E}_i \;\; \forall i \in S, t \in H \tag{a.11}$$

$$E_{i,T} \ge E_{i,0} \;\; \forall i \in S \tag{a.12}$$

$$p_{i,t}^{\mathrm{BSU}} = dis_{i,t} - ch_{i,t} + p_{i,t}^{\mathrm{s,up}} - p_{i,t}^{\mathrm{s,dn}} \;\; \forall i \in S, t \in H \tag{a.13}$$

$$q_{i,t}^{\mathrm{BSU}} = q_{i,t}^{\mathrm{s,up}} - q_{i,t}^{\mathrm{s,dn}} \;\; \forall i \in S, t \in H \tag{a.14}$$

$$(p_{i,t}^{\mathrm{BSU}})^2 + (q_{i,t}^{\mathrm{BSU}})^2 \le (\overline{S_i})^2 \;\; \forall i \in S, t \in H \tag{a.15}$$

$$0 \le \widehat{q_{i,t}^{\mathrm{s,up}}}, \widehat{q_{i,t}^{\mathrm{s,dn}}} \le \overline{S_i} \;\; \forall i \in S, t \in H \tag{a.16}$$

$$c_{i,t}^{\mathrm{s,P,up}}, c_{i,t}^{\mathrm{s,P,dn}}, c_{i,t}^{\mathrm{s,Q,up}}, c_{i,t}^{\mathrm{s,Q,dn}} \ge 0 \;\; \forall i \in S, t \in H \tag{a.17}$$

Objective function of the upper-level problem (a.1) maximizes the FSP's overall profits. The first line is associated with the DAM and RM profits of the FSP. Energy price is taken as an input (λ_t^{e}), while the upward/downward RM prices ($\lambda_t^{\mathrm{up}}, \lambda_t^{\mathrm{dn}}$) and the reserved quantities ($r_{i,t}^{\mathrm{s,up}}, r_{i,t}^{\mathrm{s,dn}}$) are obtained endogenously from the Lower-Level Problem 1 (cf. II-B). The second line in (a.1) is associated with the DLFM profit due to the provision of active and reactive power flexibility (hereinafter referred to as P-flexibility and Q-flexibility) to the DSO. The DLFM nodal active and reactive locational marginal prices (hereinafter referred to as PLMPs and QLMPs respectively) and the upward/downward P-flexibility and Q-flexibility dispatches are calculated endogenously in the clearing process of the DLFM (cf. II-C). Finally, since we consider that the DLFM follows the wholesale energy market, the active power DLFM dispatch concerning the FSP's BSUs will urge the FSP to readjust its energy market position by trading power in the Balancing Market. Thus, the last line in (a.1) represents the FSP's cost/profit from buying/selling in the BM the additional discharged/charged power (equal to the downward/upward P-flexibility provided in the DLFM by the BSUs). We assume that energy is traded in the BM at a single price ($\lambda^{\mathrm{b}}_{t,\omega}$) as in [31]. In contrast to the wholesale energy market prices (λ^{e}_{t}) which can be predicted with high accuracy [32], the BM prices are highly volatile and thus considered stochastic in this work. We tackle this uncertainty via a finite number of scenarios. Note that we consider a *risk-neutral* FSP that maximizes its expected profits. In order to explicitly address risk management and control the trade-off between profits and risk, one could use the Conditional Value at Risk (CVaR) in the objective function as a risk measure, see e.g., [33].

Constraints (a.2) and (a.3) state that the battery discharged/charged power is constrained by the battery converter's apparent power rating ($\overline{S_i}$). Binary variable $h_{i,t}$ indicates the operating mode of the BSUs, equal to 1 in the discharge mode and 0 in the charge mode (a.4). Constraint (a.5) states that the upward reserve capacity provision is constrained by the scheduled discharge/charge power traded in the energy market and the AC/DC converter's apparent power rating. The downward reserve capacity provision is constrained by the power traded in the energy market and the BSUs' power rating (a.6).

Additionally, the (upward/downward) flexibility provision to the DSO is constrained by the BSUs' apparent power rating and the energy and reserve schedules ((a.7), (a.8)). The dynamic equation of BSUs' state of charge is presented in (a.9), while constraints (a.10) and (a.11) define the BSUs' capability of upward/downward reserve capacity provisioning. Constraint (a.12) defines that at the end of the scheduling horizon, the BSUs' state of charge should be at least equal to their initial value. Each BSU is also controlled to inject/absorb reactive power. The overall active/reactive power schedules of the BSUs are presented in (a.13) and (a.14), and should be calculated such that the apparent power at each timeslot does not exceed the apparent power rating (a.15). Finally, the Q-flexibility quantity bids of the BSUs are constrained in (a.16), while nonnegativity on the flexibility market price bids is imposed in constraint (a.17). The set of optimization variables of the problem (a.1) - (a.17) is $X^{UL} = \{dis_{i,t}, ch_{i,t}, \widehat{r^{\mathrm{s,up}}_{i,t}}, \widehat{r^{\mathrm{s,dn}}_{i,t}}, \widehat{p^{\mathrm{s,up}}_{i,t}}, \widehat{p^{\mathrm{s,dn}}_{i,t}}, \widehat{q^{\mathrm{s,up}}_{i,t}}, \widehat{q^{\mathrm{s,dn}}_{i,t}}, h_{i,t}, E_{i,t}, c^{\mathrm{s,up}}_{i,t}, c^{\mathrm{s,dn}}_{i,t}, c^{\mathrm{s,P,up}}_{i,t}, c^{\mathrm{s,P,dn}}_{i,t}, c^{\mathrm{s,Q,up}}_{i,t}, c^{\mathrm{s,Q,dn}}_{i,t} | \forall i \in S, t \in H\}$.

Constraint (a.15) is linearized via a polygonal inner approximation, which we derived, described by the following set of linear constraints:

$$A_{i,m} \cdot p^{\mathrm{BSU}}_{i,t} + B_{i,m} \cdot q^{\mathrm{BSU}}_{i,t} \le \cos\left[\frac{\pi}{L}\right] \cdot \overline{S_i} \;\; \forall i \in S, t \in H, m \in [1, L] \tag{a.18}$$

where L is the number of the sides of the polygon and

$$A_{i,m} = \cos\left[\frac{(-1+2\cdot m)\cdot\pi}{L}\right], B_{i,m} = \sin\left[\frac{(-1+2\cdot m)\cdot\pi}{L}\right].$$

The case with $L = 12$ is illustrated in Fig. 3.

B. Lower-Level Problem 1: Clearing of the Reserve Market

The Lower-Level Problem 1 represents the clearing process of the reserve market, which we assume is cleared independently from the energy market. The reserve market clearing process is formulated below.

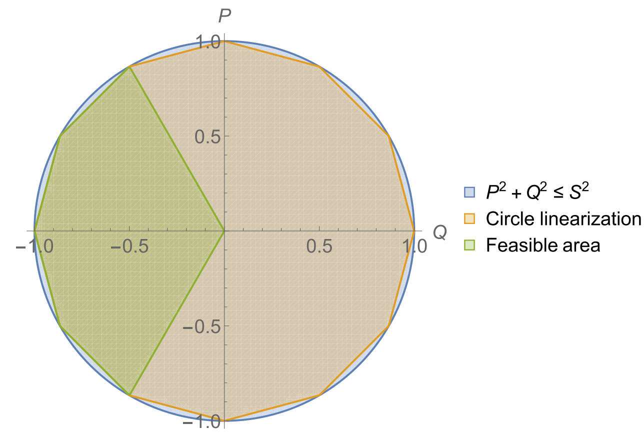

Fig. 3. Piecewise linear approximation of a circular region with a regular polygon of 12 sides.

$$\min_{X^{RM}} \sum_{t \in H} \left(\sum_{i \in G} (\tilde{c}^{\mathrm{g,up}}_{i,t} \cdot r^{\mathrm{g,up}}_{i,t} + \tilde{c}^{\mathrm{g,dn}}_{i,t} \cdot r^{\mathrm{g,dn}}_{i,t}) + \sum_{i \in S} (c^{\mathrm{s,up}}_{i,t} \cdot r^{\mathrm{s,up}}_{i,t} + c^{\mathrm{s,dn}}_{i,t} \cdot r^{\mathrm{s,dn}}_{i,t}) \right) \tag{b.1}$$

Subject to

$$\sum_{i \in G} r^{\mathrm{g,up}}_{i,t} + \sum_{i \in S} r^{\mathrm{s,up}}_{i,t} \ge R^{\mathrm{up}}_{t}; \;\; (\lambda^{\mathrm{up}}_{t}) \;\; \forall t \in H \tag{b.2}$$

$$\sum_{i \in G} r^{\mathrm{g,dn}}_{i,t} + \sum_{i \in S} r^{\mathrm{s,dn}}_{i,t} \ge R^{\mathrm{dn}}_{t}; \;\; (\lambda^{\mathrm{dn}}_{t}) \;\; \forall t \in H \tag{b.3}$$

$$0 \le r^{\mathrm{g,up}}_{i,t} \le \widehat{r^{\mathrm{g,up}}_{i,t}}; \;\; (\phi^{\mathrm{gupmin}}_{i,t}, \phi^{\mathrm{gupmax}}_{i,t}) \;\; \forall i \in G, t \in H \tag{b.4}$$

$$0 \le r^{\mathrm{g,dn}}_{i,t} \le \widehat{r^{\mathrm{g,dn}}_{i,t}}; \;\; (\phi^{\mathrm{gdnmin}}_{i,t}, \phi^{\mathrm{gdnmax}}_{i,t}) \;\; \forall i \in G, t \in H \tag{b.5}$$

$$0 \le r^{\mathrm{s,up}}_{i,t} \le \widehat{r^{\mathrm{s,up}}_{i,t}}; \;\; (\phi^{\mathrm{supmin}}_{i,t}, \phi^{\mathrm{supmax}}_{i,t}) \;\; \forall i \in S, t \in H \tag{b.6}$$

$$0 \le r^{\mathrm{s,dn}}_{i,t} \le \widehat{r^{\mathrm{s,dn}}_{i,t}}; \;\; (\phi^{\mathrm{sdnmin}}_{i,t}, \phi^{\mathrm{sdnmax}}_{i,t}) \;\; \forall i \in S, t \in H \tag{b.7}$$

Objective function (b.1) minimizes the reserve capacity procurement cost based on the market participants' reserve prices and capacity offers. The upward/downward reserve requirements are enforced in constraints (b.2) and (b.3), respectively. The dual variables of constraints (b.2) and (b.3) set the reserve up and down prices. The up and down reserve provision of the generators and the BSUs are limited in (b.4)–(b.7), based on their respecting offers. In this work, we assume that the rest of the RM participants form a competitive fringe and thus their price and quantity offers are treated as input parameters to our model. The dual variables pertaining to each constraint of the Lower-Level Problem 1 are specified at each constraint (b.2–b.7) following a semicolon. The set of the primal variables of Lower-Level Problem 1 is $X^{RM} = \{r^{\mathrm{g,up}}_{i,t}, r^{\mathrm{g,dn}}_{i,t}, r^{\mathrm{s,up}}_{i,t}, r^{\mathrm{s,dn}}_{i,t}\}$.

C. Lower-Level Problem 2: Clearing of the Flexibility Market

The proposed DLFM is a network-constrained auction-based market that is cleared solving Lower-Level Problem 2. The FSPs, either operating their own flexibility assets or acting as flexibility aggregators, submit aggregated flexibility bids, i.e. how much they can deviate from their DAM position, ($\widehat{\boldsymbol{P}^{\mathrm{s}}} := \{\widehat{p^{\mathrm{s,up}}_{i,t}}, \widehat{p^{\mathrm{s,dn}}_{i,t}}, \widehat{q^{\mathrm{s,up}}_{i,t}}, \widehat{q^{\mathrm{s,dn}}_{i,t}}; \forall i \in S, t \in H\}$, $\widehat{\boldsymbol{P}^{\mathrm{r}}} := \{\widehat{p^{\mathrm{r,up}}_{i,t}}, \widehat{p^{\mathrm{r,dn}}_{i,t}}, \widehat{q^{\mathrm{r,up}}_{i,t}}, \widehat{q^{\mathrm{r,dn}}_{i,t}}; \forall i \in F_r, t \in H\}$) and cost ($\boldsymbol{C}^{\mathrm{s}} := \{c^{\mathrm{s,P,up}}_{i,t}, c^{\mathrm{s,P,dn}}_{i,t}, c^{\mathrm{s,Q,up}}_{i,t}, c^{\mathrm{s,Q,dn}}_{i,t}; \forall i \in S, t \in H\}$ and $\tilde{\boldsymbol{C}}^{\mathrm{r}} := \{\tilde{c}^{\mathrm{r,P,up}}_{i,t}, \tilde{c}^{\mathrm{r,P,dn}}_{i,t}, \tilde{c}^{\mathrm{r,Q,up}}_{i,t}, \tilde{c}^{\mathrm{r,Q,dn}}_{i,t}; \forall i \in F_r, t \in H\}$) to the FMO. The FMO's objective is to ensure the necessary active and reactive flexibility at a minimum cost in order to address the possible contingencies (congestion and voltage issues). In other words, in case the DAM results violate the DN constraints, then the FMO will calculate the least-cost required flexibility dispatch, and the selected DERs will have to re-adjust their DAM position based on the DLFM results, in order for the DSO to secure a secure operation of its DN. The DLFM clearing process is formulated below.

$$\min_{X_{FM}} \boldsymbol{C}^{\mathrm{s}^{\mathrm{T}}} \cdot \boldsymbol{P}^{\mathrm{s}} + \tilde{\boldsymbol{C}}^{\mathrm{r}^{\mathrm{T}}} \cdot \boldsymbol{P}^{\mathrm{r}} \tag{c.1}$$

Subject to

$$\boldsymbol{0} \leq \boldsymbol{P}^{\mathrm{s}} \leq \widehat{\boldsymbol{P}^{\mathrm{s}}};\ \ (\underline{\boldsymbol{\psi}^{\mathrm{s}}}, \overline{\boldsymbol{\psi}^{\mathrm{s}}}) \tag{c.2}$$

$$\boldsymbol{0} \leq \boldsymbol{P}^{\mathrm{r}} \leq \widehat{\boldsymbol{P}^{\mathrm{r}}};\ \ (\underline{\boldsymbol{\psi}^{\mathrm{r}}}, \overline{\boldsymbol{\psi}^{\mathrm{r}}}) \tag{c.3}$$

$$\sum_{k\in\Omega_d(n)} f^{\mathrm{P}}_{nk,t} = \sum_{j\in\Omega_p(n)} f^{\mathrm{P}}_{jn,t} - d_{n,t} + g_{n,t} - ch_{n,t} + dis_{n,t}$$
$$+ p^{\mathrm{s,up}}_{n,t} + p^{\mathrm{r,up}}_{n,t} - p^{\mathrm{s,dn}}_{n,t} - p^{\mathrm{r,dn}}_{n,t};\ \ (\lambda^{\mathrm{P}}_{n,t})\ \ \forall n \in N, t \in H \tag{c.4}$$

$$\sum_{k\in\Omega_d(n)} f^{\mathrm{Q}}_{nk,t} = \sum_{j\in\Omega_p(n)} f^{\mathrm{Q}}_{jn,t} - \delta^{\mathrm{d}}_{n,t} \cdot d_{n,t} + \delta^{\mathrm{g}}_{n,t} \cdot g_{n,t} +$$
$$q^{\mathrm{s,up}}_{n,t} + q^{\mathrm{r,up}}_{n,t} - q^{\mathrm{s,dn}}_{n,t} - q^{\mathrm{r,dn}}_{n,t};\ \ (\lambda^{\mathrm{Q}}_{n,t})\ \ \forall n \in N, t \in H \tag{c.5}$$

$$U_{n,t} = U_{j,t} - 2 \cdot (r_{jn} \cdot f^{\mathrm{P}}_{jn,t} + x_{jn} \cdot f^{\mathrm{Q}}_{jn,t});\ \ (\lambda^{\mathrm{v}}_{n,t})$$
$$\forall n \in N, j \in \Omega_p(n), t \in H \tag{c.6}$$

$$\underline{V}_n \leq U_{n,t} \leq \overline{V}_n;\ \ (\underline{\psi^{\mathrm{v}}_{n,t}}, \overline{\psi^{\mathrm{v}}_{n,t}})\ \forall (n,k) \in B, t \in H \tag{c.7}$$

$$\underline{f^{\mathrm{P}}}_{nk} \leq f^{\mathrm{P}}_{nk,t} \leq \overline{f^{\mathrm{P}}}_{nk};\ \ (\underline{\psi^{\mathrm{pf}}_{nk,t}}, \overline{\psi^{\mathrm{pf}}_{nk,t}})$$
$$\forall (n,k) \in B, t \in H \tag{c.8}$$

$$\underline{f^{\mathrm{Q}}}_{nk} \leq f^{\mathrm{Q}}_{nk,t} \leq \overline{f^{\mathrm{Q}}}_{nk};\ \ (\underline{\psi^{\mathrm{qf}}_{nk,t}}, \overline{\psi^{\mathrm{qf}}_{nk,t}})$$
$$\forall (n,k) \in B, t \in H \tag{c.9}$$

Objective function of the Lower-Level Problem 2 (c.1) minimizes the flexibility procurement cost. Constraints (c.2) and (c.3) bound the DLFM dispatch of the FSP ($\boldsymbol{P}^{\mathrm{s}} := \{p^{\mathrm{s,up}}_{i,t}, p^{\mathrm{s,dn}}_{i,t}, q^{\mathrm{s,up}}_{i,t}, q^{\mathrm{s,dn}}_{i,t}; \forall i \in S, t \in H\}$) and its competitors ($\boldsymbol{P}^{\mathrm{r}} := \{p^{\mathrm{r,up}}_{i,t}, p^{\mathrm{r,dn}}_{i,t}, q^{\mathrm{r,up}}_{i,t}, q^{\mathrm{r,dn}}_{i,t}; \forall i \in F_r, t \in H\}$) based on their flexibility supply offers. As in the RM, the competing FSPs' bids are treated as parameters, we assume that they form a competitive fringe (price takers). In order to model the DN, we use the linearized DistFlow model (c.4)–(c.9) first introduced in [34]. (c.4)–(c.6) are the *branch flow* equations. In (c.4) and (c.5) the local production ($g_{n,t}$) and demand ($d_{n,t}$) are decided in the DAM, which precedes the DLFM clearing process, and thus are treated as parameters. The lower/upper limits of the square voltage magnitude ($U_{n,t}$), active power flows ($f^{\mathrm{P}}_{nk,t}$) and reactive power flows ($f^{\mathrm{Q}}_{nk,t}$) are presented in constraints (c.7)–(c.9). Potential DERs' DAM positions (i.e. parameters $d_{n,t}, g_{n,t}, ch_{n,t}, dis_{n,t}$) that require power flows violating constraints (c.7) - (c.9) will dictate the demand for flexibility. The dual variables pertaining to each constraint of the Lower-Level Problem 2 are specified at each constraint (c.2)–(c.9) following a semicolon. The PLMPs and QLMPs, at which the FSPs will be compensated for their P/Q-flexibility services, arise from the dual variables of constraints (c.4) and (c.5). These prices, taking into account the type (over/under-voltage issue or thermal line congestion), the magnitude and the location of the contingency, optimally reflect the demand for P-flexibility (PLMPs) or Q-flexibility (QLMPs). Furthermore, dual variables $\lambda^{\mathrm{P}}_{n,t}, \lambda^{\mathrm{Q}}_{n,t}$ are free variables; positive DLFM prices indicate the need for supplying power to the grid, while negative DLFM prices imply the need for absorbing power by the FSPs. As long as the DAM dispatch does not violate any constraints of the DN, then naturally $\boldsymbol{P}^{\mathrm{s}}, \boldsymbol{P}^{\mathrm{r}} = \boldsymbol{0}$ and $\lambda^{\mathrm{P}}_{i,t}, \lambda^{\mathrm{Q}}_{i,t} = 0, \forall i \in N, t \in H$. Finally, our proposed DLFM as an LP-based market satisfies the economic properties of efficiency, cost recovery and revenue adequacy [35].

III. Solution Method

The formulated non-linear bilevel problem can be recast into a Mathematical Program with Equilibrium Constraints (MPEC). To this end, we replace problems (b) and (c) with their respective Karush-Kuhn-Tucker (KKT) conditions. Note that these problems are continuous and linear, and therefore their KKT conditions are necessary and sufficient optimality conditions [36]. The resulting single-level problem contains non-linear complementarity slackness conditions. which can be linearized using the Big-M approach, as in [16] and [37]. In addition, in order to tackle the non-linearities in the objective function (a.1), we use the Strong Duality Theorem and the optimality conditions of the two lower-level problems and some algebraic operations (see [16], [37] and [38]). The resulting objective function of our single-level problem is:

$$\sum_{t\in H} \Bigg(\sum_{i\in S} (\lambda^{\mathrm{e}}_t \cdot (ch_{i,t} - dis_{i,t})) + \sum_{i\in G} (\tilde{c}^{\mathrm{g,up}}_{i,t} \cdot r^{\mathrm{g,up}}_{i,t}$$
$$+ \tilde{c}^{\mathrm{g,dn}}_{i,t} \cdot r^{\mathrm{g,dn}}_{i,t}) - R^{\mathrm{up}}_t \cdot \lambda^{\mathrm{up}}_t - R^{\mathrm{dn}}_t \cdot \lambda^{\mathrm{dn}}_t + \sum_{i\in G} (\phi^{\mathrm{gupmax}}_{i,t} \cdot \widehat{r^{\mathrm{g,up}}_{i,t}}$$
$$+ \phi^{\mathrm{gdnmax}}_{i,t} \cdot \widehat{r^{\mathrm{g,dn}}_{i,t}}) + \tilde{\boldsymbol{C}}^{\mathrm{r}^{\mathrm{T}}} \cdot \boldsymbol{P}^{\mathrm{r}} + \widehat{\boldsymbol{P}^{\mathrm{r}}} \cdot \overline{\boldsymbol{\psi}^{\mathrm{r}}} - \sum_{n\in N} (\underline{V}_n \cdot \underline{\psi^{\mathrm{v}}_{n,t}}$$
$$- \overline{V}_n \cdot \overline{\psi^{\mathrm{v}}_{n,t}}) - \sum_{(n,k)\in B} (\underline{f^{\mathrm{P}}}_{nk} \cdot \underline{\psi^{\mathrm{pf}}_{nk,t}} - \overline{f^{\mathrm{P}}}_{nk} \cdot \overline{\psi^{\mathrm{pf}}_{nk,t}}$$

$$+ \underline{f^{\mathrm{Q}}}_{nk} \cdot \underline{\psi^{\mathrm{qf}}_{nk,t}} - \overline{f^{\mathrm{Q}}}_{nk} \cdot \overline{\psi^{\mathrm{qf}}_{nk,t}}) + \sum_{n\in N} (g_{n,t} \cdot \lambda^{P}_{n,t} - d_{n,t} \cdot \lambda^{P}_{n,t}$$

$$+ dis_{n,t} \cdot \lambda^{P}_{n,t} - ch_{n,t} \cdot \lambda^{P}_{n,t} + \delta^{\mathrm{g}}_{n,t} \cdot g_{n,t} \cdot \lambda^{Q}_{n,t}$$

$$- \delta^{\mathrm{d}}_{n,t} \cdot d_{n,t} \cdot \lambda^{Q}_{n,t}) + \sum_{n\in\Omega_d(n_0)} \lambda^{v}_{n,t} - \sum_{\omega\in\Omega}\sum_{i\in S} \xi_{\omega} \cdot \lambda^{\mathrm{b}}_{t,\omega}$$

$$\cdot \left(p^{\mathrm{s,up}}_{i,t} - p^{\mathrm{s,dn}}_{i,t}\right)\Bigg) \qquad \text{(d.1)}$$

The above expression still contains bilinear terms ($dis_{n,t} \cdot \lambda^{\mathrm{p}}_{n,t}$ and $ch_{n,t} \cdot \lambda^{\mathrm{p}}_{n,t}$). This non-linearity comes from the interdependency between the DAM and the DLFM, and more specifically from constraints (c.4) and (c.5) that link decision variables from the two markets. Authors in [15] and [25] use the binary expansion technique in order to tackle the non-linearities originated from the interdependencies between two markets. In this work, we use an iterative process to deal with these non-linearities, which achieves much higher computational efficiency as will be discussed in Section IV.D. The steps of this procedure are:

1) Replace nonlinear terms $dis_{n,t} \cdot \lambda^{\mathrm{p}}_{n,t}$ and $ch_{n,t} \cdot \lambda^{\mathrm{p}}_{n,t}$ with linear terms $dis_{n,t} \cdot \overline{\lambda}^{\mathrm{p}}_{n,t}$ and $ch_{n,t} \cdot \overline{\lambda}^{\mathrm{p}}_{n,t}$, where $\overline{\lambda}^{\mathrm{p}}_{n,t}$ is a constant. This constitutes our model linear and the resulting optimization problem is a MILP.
2) Initialize the iteration counter $\mathrm{v} = 1$ and set $\overline{\lambda}^{\mathrm{p,v}}_{n,t} = 0$.
3) Solve the MILP and calculate the optimal values $\lambda^{\mathrm{p,v}^*}_{n,t}$ and the optimal objective function value ϕ^{v}. Set $\overline{\lambda}^{\mathrm{p,v}}_{n,t} = \lambda^{\mathrm{p,v}^*}_{n,t}$ and update iteration counter $\mathrm{v} = \mathrm{v} + 1$ and.
4) If $\phi^{\mathrm{v}} - \phi^{\mathrm{v\text{-}1}} \leq \epsilon$, with ϵ being a small real number, then stop the process. Otherwise, go to 3.

IV. Case Study

This section studies the performance of our proposed model using a modified IEEE 33-Bus test distribution system. The algorithm is implemented in MATLAB and in each iteration the MILP problem is solved using Gurobi 9.0.2. All simulations were performed on a personal computer with Intel Core i7 4.00 GHz and 32 GB RAM.

A. Input Data

The single-line diagram of the IEEE 33-Bus test system [34] is illustrated in Fig. 4. The total installed DG nominal capacity is 39 MW and the total base load is 18.575 MW and 11.5 MVAr. Detailed network, load and generation data of this modified system can be found in [39]. We considered two 2.5 MW x 1.6 h BSUs, located at buses 24 and 30 in the distribution network (see Fig. 4). Their discharging/charging efficiencies are set to $\eta^{\mathrm{d}}_i = \eta^{\mathrm{c}}_i = 0.93$, while the initial state of energy of the BSUs is assumed to be 87.5%. Thirteen competing FSPs are assumed to provide flexibility services to the DSO through their participation in the DLFM. These FSPs control assets that are located at buses 13, 14, 16, 17, 18, 22, 24, 25, 29, 30, 31, 32 and 33 and their active and reactive power bidding prices are set to 15 € /MWh and 3 € /MVAr, similar to [40]. Data from

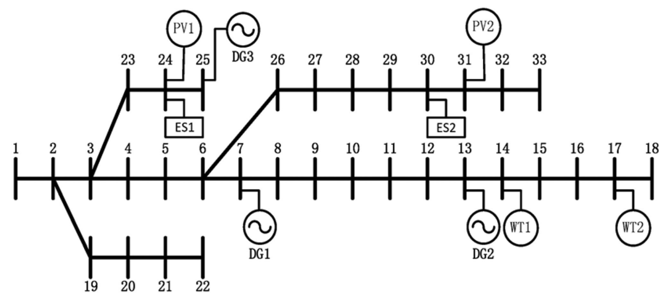

Fig. 4. IEEE 33-node distribution system.

Mavir, the Hungarian TSO [41], and the HUPX, the Hungarian Power Exchange [42], were used for the Day-Ahead Energy, Reserve and Balancing Markets with Monday, April 1 2019 as a reference date. Regarding the Reserve Market, data from the Frequency Containment Reserve market clearing process were used. Balancing Market price scenarios were formed from historical data for all Mondays of 2019 of the Mavir's Balancing Energy Market. Since the Mavir's data contains 15-minute separate up and down regulating prices, for our purposes they were transformed into hour basis and single price form using weighted average with a quantity as a weight. On the other hand, scenario weights were assigned using least distance scenario reduction technique [43] (probability of a scenario is added to the first next closest scenario, while the original scenario is removed) until only 12 scenarios remained. An interested reader can find a complete set of input data in [39]. Finally, a daily (24-h) time horizon is considered.

B. Case Study Results

To evaluate the proposed model, we examine and compare the following four cases:

1) *Case 1:* The FSP provides (energy and regulation) services to only the TSO through its participation in the DAM and RM.
2) *Case 2:* The FSP delivers flexibility services to the DSO through its participation in the DLFM. For its upward/downward P-flexibility provided to the DSO, the FSP will be paid/pay at the BM price.
3) *Case 3:* The FSP participates in all 4 markets (DAM, RM, DLFM, and BM) in a sequential manner. More specifically, the FSP initially optimizes its BSUs portfolio in order to maximize its profits from a certain market, without taking into consideration the markets that follow.
4) *Case 4:* The FSP participates in all 4 markets taking full advantage of the proposed model.

In Case 1, the FSP makes profits from providing energy and frequency regulation services to the TSO through its participation in the day-ahead energy and the reserve market, respectively. Table I illustrates the scheduling and bidding decisions of the FSP, along with the DAM and RM prices. In this case, the FSP's main target is to guarantee that the BSUs will have the maximum capacity available to offer in the RM, since this market brings the highest profits. Hence, the FSP trades energy

TABLE I
THE FSP'S SCHEDULING AND BIDDING DECISIONS, AND MARKET PRICES IN CASE 1

Hour	dis_t/ch_t (MW)	$r_t^{s,up}, r_t^{s,dn}$	$c_t^{s,up}, c_t^{s,dn}$ (€/MW)	λ_t^{e} (€/MW)	$\lambda_t^{\mathrm{up}}, \lambda_t^{\mathrm{dn}}$ (€/MW)
1	2.86	2.15,4.38	12.73, 12.73	36.09	12.73,12.73
2	-	3.65,4.38	12.73, 12.73	34.69	12.73,12.73
3	-	3.65,4.38	12.73, 12.73	35.08	12.73,12.73
4	-0.27	3.89,4.10	12.73, 12.73	34.57	12.73,12.73
5	-1.28	5,2.82	12.73, 12.73	34.75	12.73,12.73
6	-	5,2.82	12.73, 12.73	39.9	12.73,12.73
7	-	5,2.82	12.73, 12.73	49.8	12.73,12.73
8	1.55	3.45,4.61	12.73, 12.73	57.75	12.73,12.73
9	0.33	3.12,5	12.73, 12.73	58.6	12.73,12.73
10	-	3.12,5	12.73, 12.73	52.2	12.73,12.73
11	-	3.12,5	12.73, 12.73	48.81	12.73,12.73
12	-	3.12,5	12.73, 12.73	45.66	12.73,12.73
13	-	3.12,5	12.73, 12.73	45.46	12.73,12.73
14	-	3.12,5	12.73, 12.73	42.57	12.73,12.73
15	-	3.12,5	12.73, 12.73	41.92	12.73,12.73
16	-	3.12,5	12.73, 12.73	41.39	12.73,12.73
17	-2.18	5,2.82	12.73, 12.73	42.05	12.73,12.73
18	-	5,2.82	12.73, 12.73	46.02	12.73,12.73
19	-	5,2.82	12.73, 12.73	47.07	12.73,12.73
20	1.88	3.12,5	12.73, 12.73	62.41	12.73,12.73
21	-	3.12,5	12.73, 12.73	64.3	12.73,12.73
22	-0.96	3.95,4.04	12.73, 12.73	48.12	12.73,12.73
23	-0.10	4.03,3.94	12.73, 12.73	42.5	12.73,12.73
24	-2.86	6.51,1.08	12.09, 12.73	37.5	12.09,12.73

*A negative/positive value corresponds to the BSUs' charging/discharging mode.
**$dis_t = \sum_{i \in S} dis_{i,t},\ \ ch_t = \sum_{i \in S} ch_{i,t},\ \ r_t^{s,up} = \sum_{i \in S} r_{i,t}^{\mathrm{s,up}},\ \ r_t^{s,dn} = \sum_{i \in S} r_{i,t}^{\mathrm{s,dn}}$.

in the DAM mainly to gain more profit opportunities but also to precharge energy for the RM. For example, the FSP sells total power of 2.86 MW in $t = 1$, when the energy price is higher as compared to the following hours. Also, this enables the FSP to offer higher downward regulation reserve capacity. The FSP seldom performs energy arbitrage between the low-cost hours (e.g. $t = 4$ and $t = 5$) and high-cost hours (e.g. $t = 8$ and $t = 9$). In discharge hours, the FSP offers higher downward reserve capacity, while the BSUs' charging process enables it to offer higher upward reserve capacity. However, in most hours the FSP keeps its BSUs idle. The FSP's main objective is to offer high combined reserve capacity at all times (note that the upward and downward reserve prices are equal with the exception of $t = 24$), while in parallel take advantage of the most significant energy price fluctuations over time in the DAM. As shown in Fig. 5, the FSP gains 26.25 € from its participation in the DAM, and 2417.9 € from providing ancillary services to the TSO, resulting in a total profit of 2444.2 € .

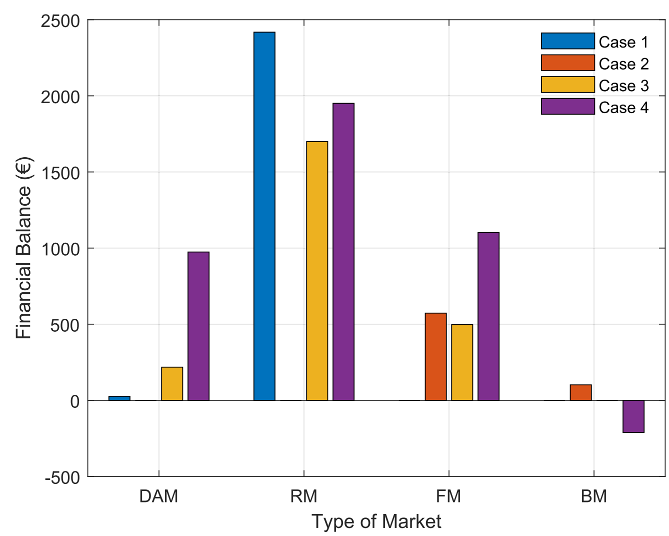

Fig. 5. Financial balance per market for all four case studies.

In Case 2, the FSP provides flexibility (upward or downward, P- or Q-flexibility services) to the DSO. For the BSUs' active power activations decided in the DLFM, the FSP will also have to pay/be paid in the BM. The purpose of the existence and operation of a DLFM is to ensure a direct participation of the DERs in the wholesale (TSO) markets without putting at risk the distribution network operation. The energy market produces a dispatch that violates several distribution network constraints at multiple hours. The FMO runs the DLFM in order for the DSO to purchase flexibility services to stabilize its network. The DLFM clearing process results are presented in Table II. In this specific case study, taking into consideration the production of the DGs and the local demand decided in the DAM, the distribution network faces mostly the over-voltage and under-voltage issues, and thus, the DSO mostly requires Q-flexibility services. Hence, we see in Table II that the BSU at node 24 draws reactive power during most of the day, when the negative QLMPs indicate the need for absorbing reactive power, while the BSU at node 30 offers reactive power in all hours (positive QLMPs). The FSP chooses only a few hours during the day to offer upward or downward P-flexibility services and using only the BSU at node 24. More specifically, the BSU at node 24 draws active power at hours $t = 11$ and $t = 15$, when the absolute value of the negative PLMP is high and, in parallel, the BM expected price is relatively low. On the other hand, the FSP chooses to discharge power at hours $t = 7$, $t = 8$ and $t = 22$ with zero PLMP, since the BM

TABLE II
THE DLFM CLEARING RESULTS IN CASE 2

Hour	$p_{1,t}^{s,up}/p_{1,t}^{s,dn}$ (MW)	$p_{2,t}^{s,up}/p_{2,t}^{s,dn}$ (MW)	$q_{1,t}^{s,up}/q_{1,t}^{s,dn}$ (MVAr)	$q_{2,t}^{s,up}/q_{2,t}^{s,dn}$ (MVAr)	$\lambda^{p}_{24,t}, \lambda^{p}_{30,t}$ (€/MW)	$\lambda^{q}_{24,t}, \lambda^{q}_{30,t}$ (€/MVAr)	$\sum_{\omega \in \Omega} \xi_\omega \cdot \lambda^{\mathrm{b}}_{t,\omega}$ (€/MW)
1	0	0	-2.18	1.70	-10.27,2.98	-6.97,3	18.59
2	0	0	-2.31	1.57	-10.27,2.98	-6.97,3	21.96
3	0	0	-2.50	1.37	-10.27,2.98	-6.97,3	24.10
4	0	0	-2.50	1.19	-10.27,2.98	-6.97,3	25.52
5	0	0	-2.50	1.28	-10.27,2.98	-6.97,3	28.33
6	0	0	-1.93	1.96	-10.27,2.98	-6.97,3	33.14
7	0.70	0	-2.21	0.39	0,11.43	-0.05,8.47	61.73
8	0.69	0	-2.22	1.33	0,12.46	-0.25,8.87	25.98
9	0	0	-2.40	1.98	-9.59,13.74	-6.88,10.34	21.63
10	0	0	-2.50	1.47	-9.59,13.74	-6.88,10.34	38.24
11	-1.63	0	-1.83	0.89	-9.54,15	-6.87,11.24	12.76
12	0	0	-1.91	2.5	-15,3.55	-10.61,3	29.81
13	0	0	-2.5	2.5	-15,3.01	-10.40,3	39.82
14	0	0	-2.34	2.5	-15,3.01	-10.40,3	41.31
15	-0.52	0	-2.29	2.5	-9.94,3.44	-6.93,3	18.71
16	0	0	-2.5	2.5	-9.97,3.41	-6.93,3	41.56
17	0	0	-2.5	1.12	-9.59,13.74	-6.88,10.34	36.79
18	0	0	-0.91	2.45	-9.64,12.02	-6.89,9.12	24.53
19	0	0	1.65	2.19	1.83,15	0.93,10.54	21.85
20	0	0	2.28	2.5	1.52,12.55	0.78,8.83	32.62
21	0	0	0.88	2.5	1.51,12.53	0.77,8.82	36.68
22	1.87	0	-1.52	2.5	0.26,12.04	0,8.69	83.26
23	-0.73	0	-2.2	1.75	0,8.55	0,6.41	54.68
24	-0.89	0	0	2.5	-10.27,2.98	-6.97,3	50.47

*A negative/positive value corresponds to downward/upward flexibility services.

TABLE III
THE BSUs POWER AND RESERVE SCHEDULES IN CASE 3

Hour	dis_t/ch_t (MW)	$r_t^{s,up}, r_t^{s,dn}$ (MW)	$p_{1,t}^{s,up}/p_{1,t}^{s,dn}$ (MW)	$p_{2,t}^{s,up}/p_{2,t}^{s,dn}$ (MW)	$q_{1,t}^{s,up}/q_{1,t}^{s,dn}$ (MVAr)	$q_{2,t}^{s,up}/q_{2,t}^{s,dn}$ (MVAr)	$\lambda_t^{\mathrm{up}}, \lambda_t^{\mathrm{dn}}$ (€/MW)	$\lambda^{p}_{24,t}, \lambda^{p}_{30,t}$ (€/MW)	$\lambda^{q}_{24,t}, \lambda^{q}_{30,t}$ (€/MVAr)
1	0	5,1.08	0	0	-1.99	2.06	12.73,12.73	-10.27,2.98	-6.97,3
2	0	5,1.08	0	0	-2.11	1.93	12.73,12.73	-10.27,2.98	-6.97,3
3	0	5,1.08	0	0	-2.37	1.71	12.73,12.73	-10.27,2.98	-6.97,3
4	-1.08	6.08,0	0	0	-1.77	2.28	12.09,12.73	-10.27,2.98	-6.97,3
5	0	5,0	0	0	-2.49	1.63	12.73,12.73	-10.27,2.98	-6.97,3
6	0	5,0	0	0	-1.74	2.32	12.73,12.73	-10.27,2.98	-6.97,3
7	0	5,0	0	0	-0.98	2.50	12.73,12.73	-10.27,2.98	-6.97,3
8	2.44	2.56,2.82	0	0	-1.99	1.99	12.73,12.73	-10.01,3.36	-6.94,3
9	5	0,8.6	0	0	0	0	12.73,12.09	-9.92,3.25	-6.93,3
10	0	0,5	0	0	-2.5	1.81	12.73,12.73	-9.54,15	-6.87,11.24
11	0	0,5	0	0	-2.5	1.14	12.73,12.73	-9.17,15	-6.82,10.85
12	0	0,5	0	0	-1.75	2.5	12.73,12.73	-15,3.54	-10.62,3
13	0	0,5	0	0	-2.4	2.5	12.73,12.73	-15,3.54	-10.62,3
14	0	0,5	0	0	-2.12	2.5	12.73,12.73	-15,3.54	-10.62,3
15	-3.60	3.12,1.40	0	0	-1.69	1.69	12.73,12.73	-9.26,15	-6.83,10.94
16	-5	7.44,0	0	0	0	0	12.09,12.73	-9.54,15	-6.87,11.24
17	0	5,0	0	0	-2.5	1.50	12.73,12.73	-9.54,15	-6.87,11.24
18	0	5,0	0	0	-0.72	2.37	12.73,12.73	-9.17,15	-6.82,10.85
19	0	5,0	0	0	1.88	2.50	12.73,12.73	1.83,15	0.93,10.54
20	2.44	2.56,2.82	0	0	0.61	1.99	12.73,12.73	1.55,12.57	0.79,8.83
21	5	0,8.60	0	0	0	0	12.73,12.09	-9.65,12	-6.89,9.11
22	0	0,5	0	0	1.40	2.5	12.73,12.73	1.02,12.07	0.52,8.67
23	-2.53	2.19,2.47	0	0	1.98	1.98	12.73,12.73	0.61,15	0.31,11
24	-5	6.51,0	0	0	0	0	12.73,12.73	0.61,15	0.31,11

*A negative/positive value corresponds to the BSUs' charging/discharging mode or downward/upward flexibility services.
**$dis_t = \sum_{i \in S} dis_{i,t}, \;\; ch_t = \sum_{i \in S} ch_{i,t}, \;\; r_t^{s,up} = \sum_{i \in S} r_{i,t}^{\mathrm{s,up}}, \;\; r_t^{s,dn} = \sum_{i \in S} r_{i,t}^{\mathrm{s,dn}}$.

prices are high enough. Overall, the FSP gains a total of 674.04 € (571.81 € from the DLFM and 102.23 € from the BM).

In Case 3, the FSP initially decides on its energy trading in the DAM ignoring the next steps (participation in RM, DLFM and BM). Then, given the BSUs' power schedule, the FSP offers reserve capacity in the RM without considering its strategy in the subsequent markets. Finally, the FSP offers its remaining power capacity to the DSO in DLFM, disregarding the forecast BM prices, at which the FSP eventually will pay/be paid its DLFM active power dispatch. Table III illustrates the final BSUs' active/reactive power schedules and reserve capacity commitments. At first, the FSP performs energy arbitrage to maximize its profit from the DAM and results in 217.67 € . This, however, hampers the BSUs' ability to offer regulation services through the RM. Comparing the RM prices in Tables I and III, we see that not co-optimizing the bidding strategies for energy and reserve

TABLE IV
THE BSUS POWER AND RESERVE SCHEDULES IN CASE 4

Hour	$dis_{1,t}/ch_{1,t}$, $dis_{2,t}/ch_{2,t}$ (MW)	$r_{1,t}^{s,up}, r_{2,t}^{s,up}$ (MW)	$r_{1,t}^{s,dn}, r_{2,t}^{s,dn}$ (MW)	$p_{1,t}^{s,up}/p_{1,t}^{s,dn}$ (MW)	$p_{2,t}^{s,up}/p_{2,t}^{s,dn}$ (MW)	$q_{1,t}^{s,up}/q_{1,t}^{s,dn}$ (MVAr)	$q_{2,t}^{s,up}/q_{2,t}^{s,dn}$ (MVAr)	$\lambda_t^{\text{up}}, \lambda_t^{\text{dn}}$ (€/MW)	$\lambda_{24,t}^{p}, \lambda_{30,t}^{p}$ (€/MW)	$\lambda_{24,t}^{q}, \lambda_{30,t}^{q}$ (€/MVAr)
1	2.5,2.5	0,0	1.61,2.5	-1.81	-0.93	-2.22	1.82	12.73,12.73	-10.36,0	-6.99,0.86
2	2.5,0	0,1.56	2.11,2.5	-2.40	0	-2.46	1.57	12.73,12.73	-10.27,2.98	-6.97,3
3	2.5,0	0,1.56	2.22,2.5	-2.78	0	-2.16	1.38	12.73,12.73	-10.27,2.98	-6.97,3
4	2.5,-0,.46	0,1.96	2.25,2.04	-2.75	0	-2.40	1.84	12.73,12.73	-10.27,2.98	-6.97,3
5	0.85,-0.67	1.65,2.54	2.39,1.37	-0.96	0	-2.45	2.22	12.73,12.73	-10.27,2.98	-6.97,3
6	0.7,-0.29	1.80,2.79	2.22,1.07	-0.98	0	-1.49	2.38	12.73,12.73	-10.27,2.98	-6.97,3
7	1.77,0	0.03,1.27	4.27,2.5	0	1.23	-1.77	1.03	12.73,12.09	-10.27,2.98	-6.97,3
8	2.5,0	0,1.56	2.14,2.5	-2.86	0	-0.66	1.47	12.73,12.73	-9.68,11.99	-6.89,9.12
9	2.5,0	0,1.56	1.51,2.5	-3.49	0	-0.95	2.02	12.73,12.73	-9.59,13.74	-6.88,10.34
10	2.5,-0.21	0,1.74	2.71,2.29	-2.29	0	-2.41	1.76	12.73,12.73	-9.59,13.74	-6.88,10.34
11	2.5,-0.88	0,2.5	0,1.41	-5	0	0	2.14	12.73,12.73	-9.26,15	-6.83,10.94
12	2.5,0.71	0,1.79	2.77,2.23	-2.23	0	-2.39	2.21	12.73,12.73	-15,3.14	-10.50,3
13	2.5,0.16	0,1.63	2.52,2.41	-2.48	0	-2.49	2.43	12.73,12.73	-15,3.01	-10.40,3
14	1.94,0	0.56,1.63	2.59,2.41	-1.85	0	-2.46	2.5	12.73,12.73	-15,3.01	-10.40,3
15	2.5,0	0,1.63	0.73,2.41	-4.27	0	-1.77	2.5	12.73,12.73	-9.94,3.44	-6.93,3
16	0.86,0	1.64,1.56	2.41,2.5	-0.86	0.07	-2.5	2.47	12.73,12.73	-9.97,3.41	-6.93,3
17	0.60,-0.80	1.90,2.25	2.11,1.7	-1.00	0	-2.34	2.17	12.73,12.73	-9.54,15	-6.87,11.24
18	2.5,-0.14	0,2.37	1.86,1.56	-3.14	0	0	2.16	12.73,12.73	-9.54,15	-6.87,11.24
19	-0.45,-0.16	2.50,2.50	1.41,1.41	0	0	2.31	2.43	12.73,12.73	1.83,15	0.93,10.54
20	0,0	2.50,2.50	1.41,1.41	0	0	2.28	2.5	12.73,12.73	1.52,12.55	0.78,8.83
21	-0.34,-0.39	2.80,2.84	1.07,1.02	0	0	2.36	2.34	12.09,12.73	1.30,15	0.66,10.75
22	-2.5,-2.5	0.61,1.73	0,0	4.35	3.27	-1.58	1.69	12.73,12.73	0.26,12.04	0,8.69
23	-2.30,-2.5	2.60,2.34	0.20,0	0	1.55	0.48	2.11	12.73,12.73	0.60,15	0.31,11
24	-0.76,-2.5	3.26,3.26	0.54,0	0	1.24	-0.07	1.98	12.09,12.73	-10.03,11.02	-6.94,8.76

*A negative/positive value corresponds to the BSUs' charging/discharging mode or downward/upward flexibility services.

leads to a reduction in the upward reserve prices during hours $t = 4$ and $t = 16$ and in the downward reserve prices during hours $t = 9$ and $t = 21$ by 5%. The lowered prices, along with the diminished available capacity to offer to the RM, reduce to a RM profit for the FSP of 1699.7 €, which is 30% lower than the profit that the FSP gains in the RM in Case 1. On the other hand, the FSP's previous scheduling and bidding decisions leave the BSUs with neither the upward nor the downward active power capacity to offer to the DSO. Thus, the BSUs provide only Q-flexibility in the DLFM, which is constrained by the maximum apparent power of the converter (Constraint a.15). Studying the DLFM QLMPs in Cases 2 and 3 (Tables II and III), we notice that the FSP, through its bidding policy, manages to increase by absolute value the DLFM prices at nodes 24 and 30 in most hours. However, the inability to provide P-flexibility services leaves the FSP earning 498 €, which is 13% lower than the FSP's profits from DLFM in Case 2. Ultimately, the *myopic* behavior of the FSP, which participates in each market disregarding the profit opportunities that follow, results in its total profit of 2415.7 €, which is 1.17% lower than in Case 1, even if the FSP participates in all four markets.

Implementation of our proposed bidding strategy, which co-optimizes the stacked revenues of the FSP coming from all four markets under study (Case 4), produces the results presented in Table IV. In this Case the FSP attempts to take advantage of all business opportunities. Fig. 5 indicates that in Case 4 the FSP achieves DAM profits far higher (974.09 €) than in Cases 1 or 3. Note that the DAM dispatch ($dis_{i,t}, ch_{i,t}$) does not determine the BSUs' state-of-charge alone, but it is only one of the two components of the final charging/discharging schedule (the other one is the DLFM active power dispatch, see (a.9), a.13). Thus, the FSP can perform arbitrage between the DAM and the DLFM (discharge in DAM and charge in DLFM and vice versa), in contrast with Cases 1, 2 and 3 where the FSP does not have this opportunity. Therefore, the FSP chooses to trade energy in the DAM much more frequently than in the previous Cases. The FSP's decision on the charging/discharging DAM schedule of the two BSUs does not consider only the DAM prices but also the profit opportunities in the RM, the nodal DLFM prices (and therefore the location of each BSU in the distribution network) and the expected BM prices. More specifically, the FSP, expecting the PLMPs at node 24 to be negative (DSO's signal that it needs downward P-flexibility in this area) during most of the day (t = 1–18, 24), uses the BSU at this node at maximum discharge power (2.5 MW) in hours t = 1–4, 8–13, 15 and 18. In this way, the FSP creates profit opportunities in the RM by maximizing its available downward reserve capacity (defined at the right-hand side of constraint a.6). However, in order for the FSP to be able to sell energy and downward regulation in the DAM and the RM respectively, the FSP has to provide downward P-flexibility to the DSO, even if it means that the FSP will have to pay for it, since the expected BM prices are higher in absolute value than the DSO's reward per unit ($\lambda_{i,t}^{\mathrm{P}}$). Hence, the FSP commits the maximum downward reserve capacity to the RM that the state-of-charge constraints of the BSU allow (constraint a.10) and the rest of the available downward power capacity is sold in the DLFM (see Figs. 6, 7).

In hours 5–7, 14, 16 and 17 the BSU at node 24 is decided to discharge power, but not at its full capacity. This produces available upward reserve capacity (defined at the right-hand side of Constraint a.5) and enables the FSP to also provide upward reserve capacity in the RM. This capacity is entirely sold in the RM, except in hour when state-of-charge constraints do not allow it (see Fig. 6). In hours 20–23 the PLMPs are positive, indicating that the DSO requires upward P-flexibility. However, constraint a.12 dictates the BSU at node 24 to charge power

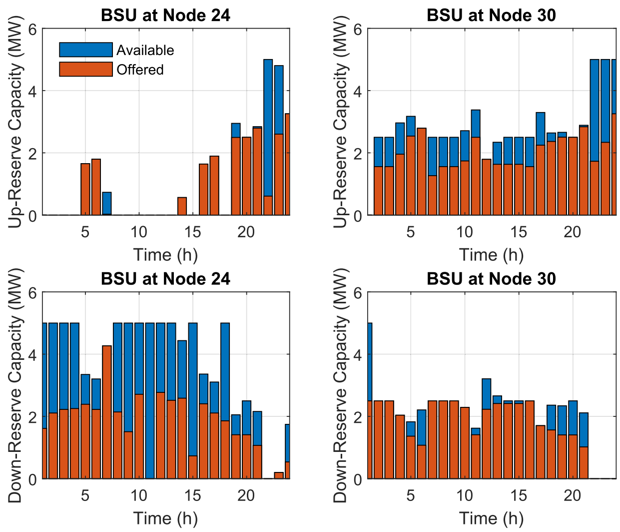

Fig. 6. BSUs' Available and Offered to the RM Reserve Capacity.

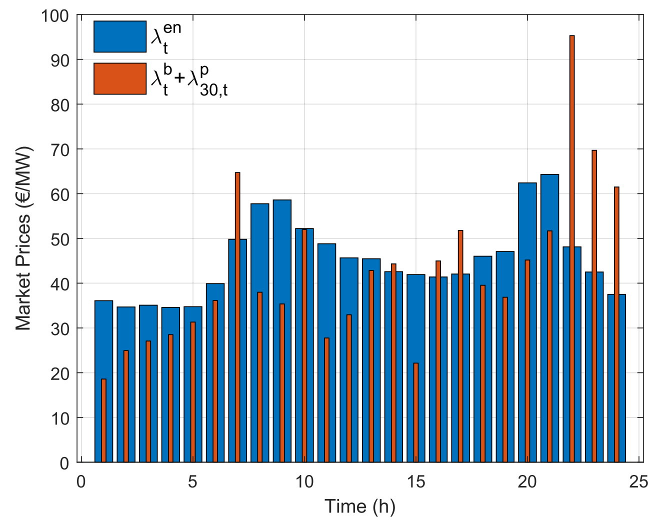

Fig. 8. Comparison between the DAM prices and the sum of the BM and the active power DLFM prices.

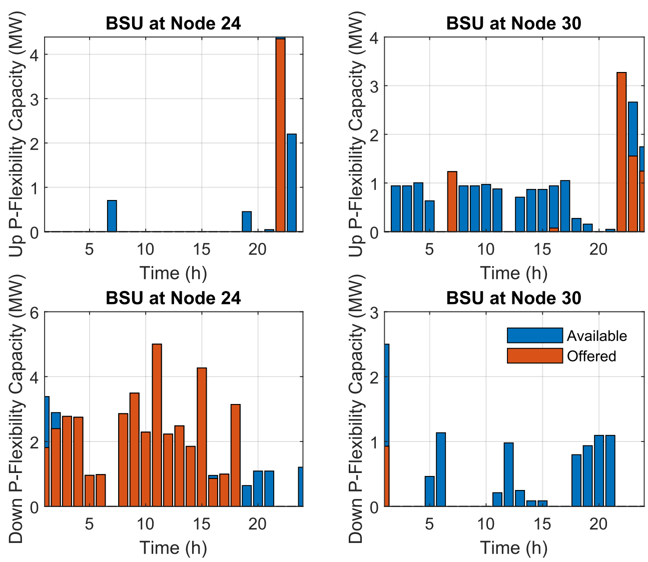

Fig. 7. BSUs' available and offered to the DLFM active power capacity.

in order to restore the state-of-charge at the end of the day. Nevertheless, in hour 22 the BM price is expected to reach its peak (83.26 €), and thus the FSP provides the DSO with 4.35 MW of upward P-flexibility, even if the DLFM price is quite low at this time.

At node 30, i.e. the location of the second FSP's BSU, the DSO requires only upward P- and Q-flexibility services throughout the day (except for the first hour, when $\lambda^p_{30,1} = 0$). In order for a BSU to be able to provide upward P-flexibility services, it should buy power in DAM. Thus, the main criterion for the FSP to decide whether the BSU will sell active power in the DLFM is the comparison between the energy price (at which the FSP will have to pay the charging power) and the sum of the PLMP at node 30 and the expected BM price (at which the FSP will be paid for the upward P-flexibility service). Therefore, the BSU at node 30 provides upward P-flexibility services to the DSO in hours 7, 16, 22, 23 and 24, when this is financially advantageous (see Fig. 8). During the rest of the day, we see in Table IV that the BSU chooses to trade power in the DAM, with the objective to have the highest possible available upward and downward reserve capacity. Hence, as shown in Fig 6, the BSU offers upward reserve capacity throughout the day and downward reserve capacity from the beginning of the day until hour 21. In the last 3 hours the high profit opportunities in DLFM and BM leads the FSP to leave no space for downward reserve capability.

Finally, throughout the day, the FSP makes profit by also providing voltage support services to the DSO, by absorbing (in hours when the QLMP is negative, $\lambda^Q_{i,t} < 0$) or supplying (in hours when the QLMP is positive, $\lambda^Q_{i,t} > 0$) reactive power to the grid. The capability of the BSUs to trade reactive power depends on their active power schedule and the apparent power rating of the converters (constraint a.15). For example, in hours 12, 13 and 14, when the absolute values of the QLMPs at node 24 are the highest throughout the day, the aggregate active power schedule of the BSU located at this node is close to zero. Therefore, the BSU can absorb reactive power at a rate very close to the maximum and increase its profits. On the contrary, in hour 11 the aggregate active power dispatch of the same BSU leaves no room for reactive power services, since it reaches the maximum apparent power potential of the BSU. At node 30, the BSU supplies reactive power the local grid at all times, as the positive QLMPs dictate.

Overall, Fig. 5 indicates that the RM profits in Case 4 are lower than in Case 1, but higher than in Case 3. In Case 1 the FSP, co-optimizing the energy and reserve services to the TSO, tries to maximize its storage capacity that is available to be offered to the TSO for regulation purposes, using the energy market. In Case 4 though, the FSP chooses not to offer its entire available capacity in the RM, since the DLFM and the BM, which chronologically follow, provide additional revenue streams. Even so, being much more active in the DAM comparing to Case 3, the FSP has

TABLE V
TOTAL PROFITS OF FSP

	Case 1	Case 2	Case 3	Case 4
FSP's Profits (€)	2444.2	674.04	2415.7	3815.5

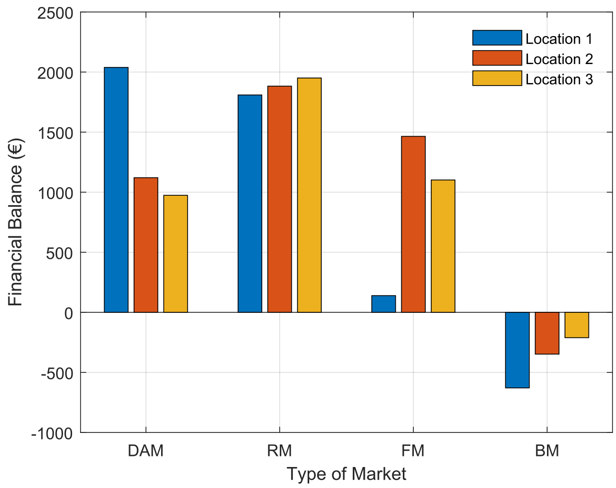

Fig. 9. Breakdown of the FSP's market revenues for each BSU location.

higher reserve potential in Case 4 and thus derives 14.7% higher RM revenues (1950.6 €). The FSP's decisions bring it profits of 1101.7 € from the DLFM, which surpass by far the FSP's profits from the local grid services in Cases 2 and 3 (higher by 92.67% and 121.22%, respectively). However, the BSUs' P-flexibility services provision to the DSO, which modify the agreed energy schedule in the DAM, lead the FSP to pay in the BM 210.94 €, in contrast with the Case 2, in which the FSP earns 102.23 € and Case 3, in which the FSP does not participate in the BM. In Table V, the aggregate FSP's profits in all four Cases are presented. Our proposed strategy achieves a total gain of 3815.5 €, which is super-linear, i.e. the revenues from jointly optimizing the BSUs' services to both the TSO and the DSO is larger than the sum of performing the individual applications (Case 1 and Case 2). In fact, the FSP earns 22.36% higher revenues in Case 4, than in Cases 1 and 2 combined. Moreover, our model (Case 4) accomplishes 57.95% higher revenues than the 'myopic' strategy of Case 3.

C. Sensitivity Analysis

This section studies sensitivity of the proposed decision-making procedure and the profitability of the FSP to some externalities, such as the location of the BSUs and the competing FSPs' offers.

1) Impact of the Location of BSUs: In this subsection, we demonstrate how the locations of the BSUs (i.e. the nodes in the DN) affect the profitability of the FSP. For this purpose, we consider three potential scenarios for the BSUs locations, namely: i) nodes 2 and 3, ii) nodes 25 and 32 and iii) nodes 24 and 30 (cf. Subsection IV-B). The FSP's individual market revenues for each location scenario are illustrated in Fig. 9. In the first scenario, the BSUs are located close to the root of the distribution grid, where the demand for flexibility, and correspondingly the DLFM prices, are low. In this case, the FSP exploits the DSO's request for downward P-flexibility, so as to perform market arbitrage and sell energy in the DAM. Thus, we observe that the DAM profits in this scenario are higher than in any other market. The second highest source of revenues for the FSP is the RM, while in the DLFM the FSP is paid only for its Q-flexibility services at a quite low price. In the BM, the FSP pays for its downward P-flexibility services. In the second scenario, the BSUs are placed at nodes 25 and 32, where the DSO's need for flexibility is rather high, rendering the DLFM much more profitable for the FSP than in other two scenarios. The BSU at node 25, since the DG3 production (see Fig. 4) mainly requires the provision of downward P-flexibility, is eligible to sell energy in the DAM during most of the day. On the other hand, the under-voltage issues at node 32 force the DSO to demand upward Q- and P-flexibility services, which leads this BSU to strategically lose money in the DAM in order to offer remunerative flexibility services to the DSO. Overall, the total revenues for the FSP are higher for location 2 (4120 €), followed by location 3 (3815.5 €) and location 1 (3358.2 €) profits.

TABLE VI
SCENARIOS OF COMPETING FSPS' PRICE OFFERS

	$c_{i,t}^{s,P,up}/c_{i,t}^{s,P,dn}$ (€/MW)	$c_{i,t}^{s,Q,up}/c_{i,t}^{s,Q,dn}$ (€/MVAr)
Scenario 1	15	3
Scenario 2	30	6
Scenario 3	45	9

2) Impact of Competing FSPs' Price Offers: In subsection IV-B we assumed that price offers of the competing FSPs are 15 € /MW for P-flexibility and 3 € /MVAr for Q-flexibility services, as in [40]. Now we study the effect the magnitude of these offers has on the results that our bidding strategy produces. To this end, we examine three scenarios of the price offers presented in Table VI. The DLFM prices in each scenario are illustrated in Fig. 10, while the individual market FSP's revenues for each scenario are presented in Fig. 11. The DLFM profits increase when increasing the competing FSP's offers since the DLFM prices rise. On the other hand, while the DAM profits in Scenario 2 are higher than in Scenario 1, they plummet in Scenario 3. This is explained by the fact that in Scenario 3 the high DLFM prices prompt the FSP to provide upward P-flexibility services to the DSO at node 30. To do that, the BSU at this node has to charge higher amounts of power in the DAM and ultimately downscale the DAM revenues. Additionally, in Scenario 3 the FSP, in contrast with Scenarios 1 and 2, makes a small profit in the BM, since the increase of the DLFM prices (and their comparison to the DAM prices) makes it profitable for the FSP to provide upward P-flexibility services, which are compensated in both the DLFM and the BM. Conclusively, the FSP in Scenarios 2 and 3 gains 30.67% and 66.57% higher profits than in Scenario 1 (i.e. 4985.8 € and 6355.5 € as compared to 3815.5 €).

D. Computational Efficiency

We evaluate the computational performance of our proposed iterative procedure using 3 case studies: a) the 15-bus radial

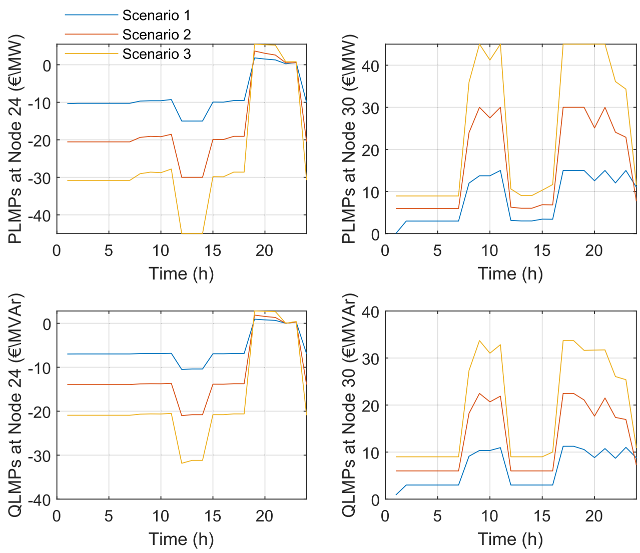

Fig. 10. DLFM prices at nodes 24 and 30 under various scenarios of competing FSPs' price offers.

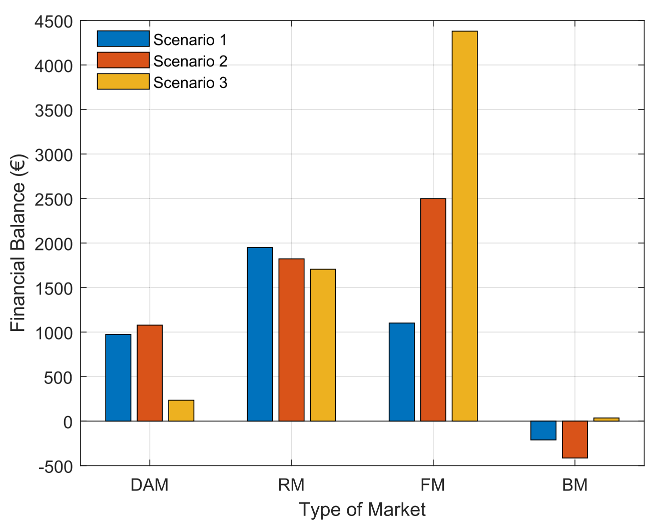

Fig. 11. FSP's individual market revenues in each price offer scenario.

TABLE VII
COMPUTATIONAL SPEED COMPARISON

	Proposed Solution Method			Binary Expansion Method		
	15-Bus	33-Bus	69-Bus	15-Bus	33-Bus	69-Bus
Iterations	3	4	4	-	-	-
Time (sec)	289	1810	3834	10000*	20000*	40000*
Profits (€)	4081.6	3815.5	2452.5	3889.8	3602.9	2159.3

*The solver reached a predefined time limit.

distribution network from [44], b) the IEEE 33-bus radial distribution system, and c) the 69-bus radial distribution system. The number of iterations and the solution times are presented in Table VII. Our algorithm terminates in 3 or 4 iterations, with each iteration requiring on average 96.3, 452.5 and 958.8 seconds, respectively, in each case study.

The binary expansion method is used as a benchmark to evaluate our solution method. In the binary expansion case, the remaining bilinear terms in (d.1) are linearized using binary approximations of variables $dis_{n,t}$ and $ch_{n,t}$, combined with additional linear constraints. In the 15-bus distribution network case study, the solver was manually stopped at 10,000 sec, achieving a sub-optimal solution (5% less profits than the proposed method), while the solver is terminated at 20,000 sec in the 33-bus network case study resulting in 6% less profits than our method. Finally, in the 69-bus distribution network the binary expansion method was terminated at 40,000 sec, resulting in 12% less profits than our proposed procedure.

V. CONCLUSION

In this paper, we considered a novel market architecture that introduces a distribution level flexibility market operating in the intra-day timeframe, between the day-ahead energy and the near-real-time balancing markets. In this context, we formulated a bilevel model for an FSP owning distributed BSUs to optimally calculate its market strategy. The bilevel problem is recast into an MPEC through a KKT-based method. An exact linearization approach, the Big-M method and an iterative process are implemented to tackle nonlinearities. Performance evaluation results demonstrate that our model achieves superlinear gains: the FSP obtains significantly higher profits through the joint optimization of both the TSO and the DSO services than the sum of the individual profits from devoting the BSUs to one of the two applications. Finally, a sensitivity analysis was conducted to showcase the impact of some externalities on the results. The proposed model can be of use to flexibility providers in the modern electricity market structure that accommodated distribution-level flexibility market. Such market is expected in the democratized and DG-rich power systems. Furthermore, our work can provide useful insights to policy makers, regulators and market operators regarding the operation of the DLFM and the TSO-DSO interaction. As a future work, we find it worthwhile to take into account uncertainties in renewable generation, load and market competition, and study the impact of the associated risks on the FSP's profitability. Also, our future research will be focused on the balancing stage, including the activation of reserves.

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