

# Near-optimal demand side management for retail electricity markets with strategic users and coupling constraints

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## ABSTRACT

The main objective of Demand Side Management (DSM) is to achieve an aggregated consumption pattern that results to energy cost reduction, welfare maximization and/or satisfaction of network constraints. This is generally pursued by encouraging electricity use at low-peak times and is a well-studied problem for the case of price-taking consumers. In this paper, however, we consider a system with strategic, price-anticipating consumers with private preferences that choose their electricity consumption patterns so as to maximize their own benefit. In this context, we take on the problem of coordinating the strategic consumers' consumption behavior so as to satisfy system-wide constraints without sacrificing their welfare. To do so, we draw on concepts of indirect mechanism design and propose a novel DSM architecture that is able to bring the system to Nash Equilibrium. The proposed scheme preserves both the budget-balance and the individual rationality properties. According to our evaluation, the proposed DSM architecture achieves a close to optimal allocation (1%–3% gap), compared to an “optimal” system that would use central optimization of user loads without user consensus or protection of their privacy.

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## 1. Introduction

Demand Response, or Demand Side Management (DSM), refers to the idea of driving electricity consumption at times when it is more efficient to serve it. As analyzed in [1], residential participation in DSM is commonly envisaged via aggregated participation because of implementation and scalability issues. An Electricity Service Provider (ESP) is considered for the role of aggregating and coordinating the users' actions.

Naturally, each user is typically trying to optimize his/her own objective, which may or may not be in line with the social objective. A particular stream of game theory called mechanism design is essentially the tool for designing rules for systems with strategic participants who hold private information. In our case, these rules consist of an allocation rule, through which end users determine their consumption pattern, and a billing rule, through which their bills are determined. The aim of mechanism design is that the system at equilibrium exhibits good performance guarantees.

## 2. DSM system requirements

We start by describing in detail the objectives that we set for a DSM mechanism. According to our requirement analysis [2], a DSM architecture has to fulfill three main properties:

### 2.1. Welfare maximization

The first property is the maximization of the welfare (i.e. the aggregated users' utility). The utility of a user/energy consumer is defined as the difference between: (i) a metric (noted here as valuation function) that quantifies in monetary terms how much the user values/appreciates a specific energy consumption profile/pattern and (ii) the bill that the user pays for it.

Making strong assumptions on the form of user's preferences renders the system conducive to theoretically strong results, but the validity of these assumptions is often questionable [3]. Also, a common assumption made regarding the user's behavior is that the user is modeled as a price-taker, which means that the effect of his/her own decisions on the electricity price is negligible and not accounted for. While this might be reasonably accurate for large systems, in emerging energy communities and decentralized systems this assumption is no longer valid, as the user may be a price-anticipator. The latter user model only makes

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things more complicated when it comes to welfare maximization and it is avoided in most of the literature (see [4] and references therein). In contrast, in the present paper we consider users who are price anticipators.

An additional typically desired property is that of *individual rationality*. A mechanism is called individually rational if each and every user benefits from participating in it. In other words, at equilibrium, each user should be better-off by participating in the DSM, rather than by not participating.

## 2.2. Budget-balance

Budget-balance is a generally required property for a market mechanism [5] and, in our context, refers to the requirement that the mechanism should not subsidize the DSM participation nor should it extract a surplus. Rather, the sum of payments from paying participants must equal the sum of the system's cost plus profits of the service providers [6]. We consider a model where the ESP imposes a fixed profit percentage on the units traded. Indicative use cases of this business model are: (i) the case of energy cooperatives [7], (ii) public companies [8] acting as ESPs, (iii) private monopolistic companies with regulated profit margins, (iv) virtual associations of users [9], (v) islanded energy communities.

## 2.3. Constraint satisfaction

The third property for a DSM mechanism is the coordination of the aggregated users' consumption so as to satisfy system-wide constraints. These constraints indicatively aim to:

- Case (a) keep the aggregated consumption below a certain threshold at all times, or
- Case (b) keep the system's overall cost within a certain range.

The necessity of satisfying such constraints is met in many use cases of modern smart grids, which may aim at:

- (1) Enhancing the self-sufficiency of the community
- (2) Keeping islanded microgrids economically viable [10]
- (3) Mitigate suppliers' exercise of market power by taking coordinated action to reduce the demand in the face of such situations [11]
- (4) Meeting the physical network's constraints by implementing the DSO's orders
- (5) Enhancing the community's participation in flexibility markets [12,13]
- (6) Reducing CO<sub>2</sub> emissions and respecting modern legal frameworks towards energy cost reduction [14]
- (7) Enhancing RES penetration by adapting demand to the intermittent generation [15]

From a technical point of view, satisfying a system-wide constraint can be a challenge. In particular, constraint satisfaction typically depends on the aggregated consumption profile of end users. This couples the system's decision variables that are controlled by different users, creating dependencies among them, and bringing a fair amount of complications in the underlying *n*-person game [16]. The DSM architecture that we propose can be used for both cases (a) and (b) of constraints described above. In this paper, we provide a theoretical analysis for case (b), which is the most difficult of the two to address, but in the evaluation section we present simulation experiments for both cases.

Furthermore, in cases where only the operational constraints of the loads are considered (e.g. maximum power consumption of a device), curtailing a load during a Demand-Response event for a certain timeslot (like for example in [1]) often has a rebound

effect that results in a higher-than-expected consumption in later timeslots. In contrast, in this work we model time-coupling constraints (that apply the whole scheduling horizon) so that the rebound effect is also taken into account and mitigated.

Further requirements may also apply for the DSM mechanism, depending on the context and the particular business model of the system. Designing a DSM architecture that exhibits properties tailored to each specific business model is an open research topic. In the context of SOCIALENERGY EU project [17], we design allocation and billing rules for various settings, including real-time consumption curtailments ([18,19]) and day-ahead consumption scheduling ([20] and the present paper). In the current paper, we focus only on period-ahead scheduling. Various other studies have proposed solutions for aligning the user's day-ahead consumption schedule with his/her updated real-time needs (see [6, 21–23]), and they can be applied in combination with the scheme proposed in this paper.

The main contribution of the present paper is the design of a DSM architecture for an environment with price-anticipating users. Our proposed mechanism is the first to address all three aforementioned requirements simultaneously: (a) achieving a close to optimal allocation, (b) keeping the system budget-balanced and (c) providing the ESP with the capability of (indirectly) controlling the aggregated consumption so as to meet system-wide constraints (e.g., energy cost reduction). We will rigorously prove these properties to hold theoretically for our proposed DSM mechanism, and we also verify them experimentally in the performance results section. The proposed DSM scheme preserves the individual rationality property and does not compromise the user's privacy.

## 3. Related work and comparison of approaches

In the DSM context described above, we are interested in mechanisms that satisfy the three requirements that were set in the previous section. The welfare-maximizing requirement is highly dependent on user modeling. That is, a theoretically optimal allocation can be achieved only under certain assumptions on the users' preferences representation. DSM studies can be categorized into three main branches with respect to the way they model user preferences.

The first research branch includes many works (e.g., [23–33]) that consider users who need to consume a certain amount of energy within a certain period, for example, charge their electric vehicle. These users have no preferences regarding how much they consume at each timeslot as long as their state of charge is met upon their deadline.

The second research branch in the literature (e.g., [1] and [34–40]) considers specific user preference models and price sensitive consumption patterns. In particular, the study in [34] approaches the problem with a regret-based algorithm, the work in [36] uses Simulated Annealing, while the rest of the works typically formulate a convex optimization problem and reach the optimal solution by solving its dual problem. During this process, the ESP and the users solve their local problems and exchange messages with intermediate results. Under the assumption of price-taking users, the final allocation is welfare-maximizing.

In the third branch of research (e.g., [4,41]) the user as a price-taker assumption is relaxed, and users are considered to be price-anticipators. This means that the users consider the effect of their own actions on the prices. In this setting, the dual approach no longer achieves welfare maximization, as analyzed in [42], chapter 21. Instead of solving the dual problem, the studies in this third branch opt for a Vickrey–Clarke–Groves (VCG) mechanism that has the additional desired (in this setting) property of incentive compatibility. This means that users can only lose

through untruthful bidding, thus leaving no room for cheating. However, the practical applicability of the VCG mechanism is highly debated because it is a *direct* mechanism (i.e., requiring users to reveal their preferences to the ESP), which raises not only privacy but also representation issues (see [6] for a more detailed analysis). In contrast, in the present work we opt for an *indirect* mechanism, in the sense that the ESP does not control users' load directly but users decide upon their own allocation after exchanging messages with the ESP via an iterative pricing scheme.

From the above three user model research branches, only the first one preserves the *budget-balance* property. The dual decomposition approach typically ends up with the market-clearing prices and extracts a big surplus from the users, especially when the latter are price-takers. Also, the VCG mechanism is inherently not *budget-balanced*.

Finally, constraint satisfaction complicates things when it comes to indirect mechanisms. This is because typical market-clearing approaches are often not suitable for constraint satisfaction, especially when the constraints couple the optimization variables. Thus, the works that induce some kind of controllability, either relax the welfare-maximization requirement [43] or the user preferences modeling [44], or they adopt a centralized optimization approach [45,46] with a consequent assumption of direct control on user loads.

In this work we present a DSM architecture for price-anticipating users that: (i) achieves near optimal welfare (reaches 91%–99% of the optimal value), (ii) is theoretically proven to preserve the *budget balance* and the *individual rationality* properties, (iii) provides the ESP with controllability over the overall system's cost (which is a coupling and quadratic constraint). To the best of our knowledge, this is the first DSM mechanism that satisfies all three requirements described above. Moreover, we achieve this via an *indirect* mechanism that does not compromise the user's privacy and sense of deliberate choice.

#### 4. System model

We consider an electricity market comprised of an ESP and a set  $N \triangleq \{1, 2, \dots, n\}$  of self-interested consumers, hereinafter referred to as users. We also consider a discrete representation of time, where continuous time is divided into timeslots  $t \in H$ , of equal duration  $s$ , with  $H \triangleq \{1, 2, \dots, m\}$  representing the scheduling horizon. A user possesses a number of controllable appliances where each appliance bears an energy demand. For ease of presentation and without loss of generality, we consider each appliance as one user and will use the terms "user" and "appliance" interchangeably. It is of course possible to include many appliances of the house in the DSM scheme. If the consumptions of different appliances are de-coupled (independent of each other), they can participate in the DSM as different users (their bills will add up to the bill of the actual user). If their operation is not independent, the user participates via a Home-Energy-Management-System (HEMS) that coordinates the consumption of different appliances. The proposed system model is valid for both of the above cases.

##### User & Appliance modeling

An appliance  $i$  requires an amount of energy for operation. For example, if the appliance's operating power is 1 W and  $s = 1$  h, then the energy that the appliance consumes in one timeslot of operation is 1 Wh. This energy consumption is controllable via a decision variable  $x_i^t$  that denotes the amount of energy consumed by appliance  $i \in N$  during timeslot  $t \in H$ . Throughout this paper, we assume  $x_i^t \geq 0$ . Each appliance  $i$  is characterized by

- (i) a feasible consumption set, defined by a set of constraints on  $x_i^t$ , which is presented below and

- (ii) a valuation function of the energy that  $i$  consumes throughout  $H$ .

The aforementioned set of constraints includes upper and lower consumption bounds, restrictions on consumption timeslots and a coupling constraint. More specifically, appliance  $i$  cannot consume at any time more than an upper bound  $\bar{x}_i$ , that is,

$$0 \leq x_i^t \leq \bar{x}_i, \quad (1)$$

for all  $i \in N$  and  $t \in H$ . An appliance  $i$  also bears a set of timeslots  $h_i \subseteq H$ , in which its operation is feasible (e.g., an electric vehicle can be plugged in only at timeslots during which its owner is at home):

$$x_i^t = 0, \quad t \notin h_i. \quad (2)$$

We denote an appliance's feasible consumption profile by a vector  $\mathbf{x}_i = \{x_i^1, x_i^2, \dots, x_i^m\} \in \mathcal{X}_i$ , where  $x_i^t$  satisfies Eqs. (1) and (2). We denote by  $\mathcal{X}_i \subseteq \mathbb{R}^m$  the feasible set for  $i$ 's consumption profile:

$$\mathcal{X}_i \triangleq \{\mathbf{x}_i \mid x_i^t \text{ such that (1), (2) hold}\}, i \in N. \quad (3)$$

Note that these intervals do not need to be accurate. They can also be statistical approximations without any user input if desired. In this case the user's control over his/her appliances is ensured via a classical manual controller that overrules the appliances DSM controller. Thus, the DSM scheduling process can only benefit the user without requiring the latter to have perfect forecasts or relinquish control.

Finally, the  $n \times m$  matrix containing all users' consumptions at all timeslots is denoted as  $X = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \in \mathcal{X}$ , where  $\mathcal{X} = \mathcal{X}_{i \in N}$  stands for the Cartesian product of the  $\mathcal{X}_i$ 's.

The valuation function is expressed in monetary units (\$), and it is private (the user does not share it with the ESP or other users). It is generally a function of  $\mathbf{x}_i$  and expresses the maximum amount of money that a user is willing to pay for the operation profile  $\mathbf{x}_i$ . The valuation function  $v_i(\mathbf{x}_i)$  can take various forms, depending on the appliance. Let  $\mathbf{0}^m$  denote the  $m$ -vector with all of its elements equal to zero. We adopt some common assumptions ([1,4,35–40]) based on microeconomics theory [47] on the form of  $v_i(\mathbf{x}_i)$ :

**Assumption 1.** Zero consumption brings zero value to the user:

$$v_i(\mathbf{0}^m) = 0 \quad (4a)$$

**Assumption 2.** Consuming more does not make the user less happy. That is, for two arbitrary vectors  $\mathbf{x}_{iA}$  and  $\mathbf{x}_{iB}$ , we have:

$$v_i(\mathbf{x}_{iA}) \leq v_i(\mathbf{x}_{iA} + \mathbf{x}_{iB}), \quad \forall \mathbf{x}_{iA}, \mathbf{x}_{iB}. \quad (4b)$$

**Assumption 3 (Concavity).** For two arbitrary vectors  $\mathbf{x}_{iA}$ ,  $\mathbf{x}_{iB}$  and for any scalar  $0 < a < 1$ , we have:

$$av_i(\mathbf{x}_{iA}) + (1 - a)v_i(\mathbf{x}_{iB}) \leq v_i(a \cdot \mathbf{x}_{iA} + (1 - a) \cdot \mathbf{x}_{iB}). \quad (4c)$$

Finally, we define the user's utility as the difference between the user's valuation for his/her consumption profile and the bill he/she has to pay for it:

$$U_i(\mathbf{x}_i) = v_i(\mathbf{x}_i) - b_i(\mathbf{x}_i). \quad (5)$$

##### System Cost & Electricity Billing

The ESP is responsible for purchasing energy from the wholesale market and delivering it to the users. We assume that the ESP faces a per-timeslot cost that is a strictly increasing function  $C^t(\cdot)$  of the aggregated consumption  $\sum_{i \in N} x_i^t$ . In particular, quadratic or piecewise linear functions are widely used in the state-of-the-art

literature [23–40], to model the generation cost of marginal units as explained in [48]. In our performance results section we use a quadratic cost of the form

$$C^t \left( \sum_{i \in N} x_i^t \right) = c \cdot \left( \sum_{i \in N} x_i^t \right)^2. \quad (6)$$

This choice of the cost function was made only for the sake of being specific, and any increasing and convex function could have been used instead.

As explained in the Introduction, we consider a use case where the ESP needs to be able to control the system's cost so as to keep it below a certain threshold  $C_{ref}$ . We assume that, over the scheduling horizon  $H$ , the ESP faces a financial cost

$$C = \sum_{t \in H} C^t \left( \sum_{i \in N} x_i^t \right). \quad (7)$$

The ESP applies a billing rule  $b(\mathbf{x}_i)$  to each user, which has to satisfy the following requirements that we have set for the DSM system:

**Requirement 1.** The sum of the users' bills should add up to the system's cost plus the ESP's profit:

$$\sum_{i \in N} b(\mathbf{x}_i) = (1 + \pi) \cdot C^t \left( \sum_{i \in N} x_i^t \right) \quad (8)$$

where  $\pi$  is the ESP's profit factor. Eq. (8) captures the *budget balance* property analyzed in the introduction. Note, that, in this paper, we do not discuss the wholesale market architecture and how the ESP participates in it. Exactly for this reason, in our model we kept the ESP's profit margin  $\pi$  constant, leaving open any further discussion on the wholesale market architecture.

**Requirement 2.** At equilibrium, each user should have weakly positive utility.

This is equivalent to stating that each user would be better-off participating in the mechanism rather than not participating in it. This is equivalent to the *individual rationality* property.

#### ESP-user interaction & implementation

We assume a communication network, built on top of the power grid, allowing the ESP and the users to exchange messages. In particular, in order for an indirect mechanism to be implemented, we assume that the users can respond to demand queries. In particular, the ESP provides the user with the necessary billing data and the user is expected to respond with his/her demand, that is, with the desired consumption vector  $\mathbf{x}_i$  that maximizes the user's utility  $U_i(\mathbf{x}_i)$  given in Eq. (5).

Since an efficient allocation involves a certain degree of coordination among users, it may take several message exchanges between the ESP and each user to converge to equilibrium. Therefore, for a user to be able to respond to each demand query in a reasonable amount of time, the billing rule should be simple enough, corresponding to a computationally tractable problem. A sufficient condition that fulfills this property is captured in a third requirement, which is:

**Requirement 3.** The user's bill  $b(\mathbf{x}_i)$  is convex in  $\mathbf{x}_i$ .

To justify the sufficiency of **Requirement 3**, recall the definition of the user's utility from Eq. (5). The first term is concave by **Assumption 3**. A convex  $b(\mathbf{x}_i)$  makes the user's utility  $U_i(\mathbf{x}_i)$  concave in  $\mathbf{x}_i$ . Thus, the user's response to a demand query becomes a convex optimization problem, which is tractable.

## 5. Problem formulation

In this section, we formalize the problem to be solved, which is maximizing the aggregated users' utility:

$$\max_{\mathbf{x}_i \in \mathcal{X}_i, i \in N} \sum_{i \in N} (v_i(\mathbf{x}_i) - b(\mathbf{x}_i)) \quad (9)$$

while keeping the system's cost below the predefined threshold  $C_{ref}$ . By substituting Eq. (8) in Eq. (9), the problem becomes equivalent to:

$$\max_{\mathbf{x}_i \in \mathcal{X}_i, i \in N} \left\{ \sum_{i \in N} v_i(\mathbf{x}_i) - (1 + \pi) \sum_{t \in H} \left[ C^t \left( \sum_{i \in N} x_i^t \right) \right] \right\} \quad (10)$$

$$s.t. \sum_{t \in H} \left[ C^t \left( \sum_{i \in N} x_i^t \right) \right] \leq C_{ref}. \quad (10a)$$

Note that constraint (10a) couples the variables  $x_i^t$  across both  $i \in N$  and  $t \in T$ . In the following lemma, we demonstrate that this is a standard convex optimization problem where a concave function is maximized over a convex set  $\mathcal{X} \subseteq \mathbb{R}^{n \times m}$  defined by the inequality constraints (1), (10a) and the equality constraint (2).

**Lemma 1.** The problem defined by Eq. (10) under constraints (1), (2) and (10a) is a convex optimization problem. In particular:

(i) The objective function

$$f(X) = \sum_{i \in N} v_i(\mathbf{x}_i) - (1 + \pi) \cdot \sum_{t \in H} \left[ C^t \left( \sum_{i \in N} x_i^t \right) \right]$$

is concave in  $X \in \mathcal{X}$ .

(ii) Inequality constraints (1) and (10a) are convex in  $X \in \mathcal{X}$ .

(iii) Equality constraint functions (2) are affine in  $X \in \mathcal{X}$ .

**Proof.** (i) Since  $\sum_{i \in N} v_i(\mathbf{x}_i)$  is a sum of concave functions in subspaces of  $\mathcal{X}$ , it is concave in  $\mathcal{X}$ . Let  $\mathbf{1}_n$  be the all-ones  $n$ -dimensional vector and  $\mathbf{1}_{n \times n}$  the all-ones  $n \times n$  dimensional matrix. Let also  $\mathbf{x}^t \triangleq (x_1^t, x_2^t, \dots, x_n^t)^T$  be the vector containing all the users' consumptions during timeslot  $t$ . Then

$$C^t \left( \sum_{i \in N} x_i^t \right) = c \cdot ((\mathbf{1}_n)^T \cdot \mathbf{x}^t)^2 = c \cdot (\mathbf{x}^t)^T \mathbf{1}_{n \times n} \mathbf{x}^t$$

is convex, because it is a quadratic function and  $\mathbf{1}_{n \times n}$  is positive semi-definite. Therefore,  $-\sum_{t \in H} [C^t(\sum_{i \in N} x_i^t)]$  is concave in  $\mathcal{X}$ , as a sum of concave functions in subspaces of  $\mathcal{X}$ . This completes the proof of (i), because  $f$  is a sum of concave functions.

(ii) Constraint (1) is trivially convex and (10a) is also convex as shown in the second term of  $f$ .

(iii) Constraint (2) is trivially affine, for all  $i \in N$ , in a subspace of  $\mathcal{X}$ , and so it is also affine in  $\mathcal{X}$ . ■

Thus, the optimization problem given by (10) is convex and has a global optimal solution. If the valuations  $v_i(\mathbf{x}_i) \triangleq v_i(x_i^t; t \in H)$  were known, it could be solved through the use of an interior point method. However, the functions  $v_i(\mathbf{x}_i)$  are private to each user  $i$ . Moreover, we assume strategic users who opt for maximizing their own utility, that is, each user chooses his/her consumption vector according to

$$\mathbf{x}_i = \operatorname{argmax}_{\mathbf{x}_i \in \mathcal{X}_i} \{v_i(\mathbf{x}_i) - b(\mathbf{x}_i, \mathbf{x}_{-i})\}, \quad (11)$$

where  $\mathbf{x}_{-i}$  are the consumption vectors of other users. The latter objective of the individual users depends on the billing rule  $b(\mathbf{x}_i, \mathbf{x}_{-i})$  and is not necessarily aligned with the social objective. Since the cost function couples the users' variables, a user's utility

depends not only on her/his own profile  $\mathbf{x}_i$ , but also on the other users' consumption choices  $\mathbf{x}_{-i}$ . This latter fact brings problem (10) in the realm of game theory. In order to bring the system to an equilibrium that optimizes (10), we will draw on the concepts of mechanism design.

We consider a game-theoretic framework, where the ESP announces the billing rule and users iteratively select their preferred allocations, resulting in the following game:

*Definition of game  $\Gamma$ :*

- *Players:* users in  $N$
- *Strategies:* each user selects  $\mathbf{x}_i$ , according to (11)
- *Payoffs:* a user's payoff is his/her utility as defined in (5)

In what follows, we opt for designing a DSM architecture that consists of:

- (a) an *indirect and individually rational* mechanism, including a *budget-balanced* billing rule, which is implemented in best-response strategies. Although we have to relax the welfare-maximization property, we will actually be able to reach a near-optimal solution.
- (b) an algorithm at the ESP side, which iteratively decides a parameter of the billing rule. This will provide the ESP with online controllability over the system's cost, so that constraint (10a) is satisfied at equilibrium.

## 6. Proposed DSM architecture

In this section, we describe the proposed DSM architecture that fulfills the aforementioned requirements. The description is complemented with the presentation of theorems proving analytically that the requirements we have set for the overall DSM system are actually fulfilled.

### 6.1. The billing rule

At any iteration, each user  $i$  chooses his/her strategy assuming the strategies  $\mathbf{x}_{-i}$  of other users to be constant. Thus, from a user's perspective, at a certain iteration, his/her bill only depends on his/her own choice of  $\mathbf{x}_i$ . The following equation presents the proposed billing rule:

$$b(\mathbf{x}_i) = \sum_{t \in H} \left[ x_i^t \cdot \frac{(1 + \pi) \cdot C^t \left( \sum_{j \in N} x_j^t \right)}{\sum_{j \in N} x_j^t} \right] + \gamma \cdot \left[ \sum_{t \in H} \left[ x_i^t \cdot \sum_{j \neq i} (x_j^t) \right] - \frac{\sum_{j \in N} \left[ \sum_{t \in H} \left( x_j^t \cdot \sum_{k \neq j} x_k^t \right) \right]}{n} \right]. \quad (12)$$

The first term of the sum in Eq. (12) is identical to existing billing rules that charge a user based on the cost of his/her part of the consumption. The second term has the purpose of rewarding/penalizing flexibility/inflexibility (i.e., the ability of user  $i$  to modify his/her energy consumption profile). The value of  $\gamma$  is iteratively updated by the ESP. The rationale of Eq. (12) is that it penalizes users who synchronize their loads with others, and uses the applied penalties to reward users who counter-balance the aggregated consumption by consuming their load at off-peak timeslots. With respect to the billing rule, we state the following lemma:

**Lemma 2.** For constant values of  $\mathbf{x}_{-i}$ , the bill  $b(\mathbf{x}_i)$ , given by Eq. (12), is strictly convex in  $\mathbf{x}_i$ .

**Proof.** We denote by  $\mathcal{H}^b$  the Hessian matrix of the billing function  $b(\mathbf{x}_i)$ , defined in Eq. (12). We have to show that  $\mathcal{H}^b$  is positive definite.

Substituting Eq. (6) in Eq. (12), we get

$$b(\mathbf{x}_i) = (1 + \pi) \sum_{t \in H} \left[ x_i^t \cdot c \cdot \left( \sum_{j \in N} x_j^t \right) \right] + \gamma \cdot \left[ \sum_{t \in H} \left[ x_i^t \cdot \sum_{j \neq i} (x_j^t) \right] - \frac{1}{n} \cdot \sum_{j \in N} \left[ \sum_{t \in H} \left( x_j^t \cdot \sum_{k \neq j} x_k^t \right) \right] \right] = (1 + \pi) \sum_{t \in H} \left[ x_i^t \cdot c \cdot \left( \sum_{j \in N} x_j^t \right) \right] + \gamma \left[ \frac{n-1}{n} \cdot \sum_{t \in H} \left[ x_i^t \cdot \sum_{j \neq i} (x_j^t) \right] - \frac{1}{n} \cdot \sum_{j \neq i} \left[ \sum_{t \in H} \left( x_j^t \cdot \sum_{k \neq j} x_k^t \right) \right] \right].$$

By taking the partial derivatives, we obtain

$$\frac{\partial b(\mathbf{x}_i)}{\partial x_i^{t_1}} = (1 + \pi) c \sum_{j \in N} x_j^{t_1} + (1 + \pi) c x_i^{t_1} + \gamma \left[ \frac{n-1}{n} \sum_{j \neq i} (x_j^{t_1}) - \frac{1}{n} \sum_{j \neq i} x_j^{t_1} \right],$$

and

$$\mathcal{H}_{t_1 t_2}^b = \frac{\partial^2 b(\mathbf{x}_i)}{\partial x_i^{t_2} \partial x_i^{t_1}} = \begin{cases} 2(1 + \pi)c, & t_1 = t_2 \\ 0, & t_1 \neq t_2. \end{cases}$$

Thus,  $\mathcal{H}^b = \text{diag}(2(1 + \pi)c)$  is positive definite. ■

The user communicates his/her demand profile  $\mathbf{x}_i$  to the ESP and receives the respective bill  $b(\mathbf{x}_i)$ . Since problem (11) is convex (by Lemma 1 and Assumption 3), the user can apply a gradient projection method to compute his/her best response. Thus, the user does not need to know the strategies  $\mathbf{x}_{-i}$  of other users and privacy is preserved. Next, we analyze the properties of game  $\Gamma$ :

**Theorem 1.** A Nash Equilibrium (NE) for game  $\Gamma$  (i) always exists and (ii) is unique. Furthermore, (iii) best-response dynamics converges to the Nash Equilibrium strategy vector.

**Proof.** (i) The user's payoff is his/her utility, given by Eq. (5). The first term is concave in  $\mathbf{x}_i$  by Assumption 3. The second term is strictly convex in  $\mathbf{x}_i$  by Lemma 1. Hence, for  $x_i^t \geq 0$ ,  $U_i(\mathbf{x}_i)$  is strictly concave in  $\mathbf{x}_i$ . Since this holds for every user  $i$ ,  $\Gamma$  is a strictly concave  $n$ -person game. Thus, by [49, th.1], we conclude that a NE exists.

(ii) Based on [50], it suffices to show that  $\Gamma$  is an exact potential game with a concave potential function. Towards finding a potential, we consider the function

$$p(X) = \sum_{i \in N} v_i(\mathbf{x}_i) - \sum_{t \in H} \left[ \frac{(1 + \pi)c}{2} \cdot \left( \sum_{i \in N} x_i^t \right)^2 \right]$$

$$+ \sum_{t \in H} \left[ \frac{(1 + \pi)c}{2} \cdot \sum_{i \in N} [(x_i^t)^2] + \frac{\gamma(n-2)}{2n} \cdot \sum_{i \in N} \left( x_i^t \cdot \sum_{j \neq i} (x_j^t) \right) \right].$$

Function  $p(X)$  has the property of a potential:

$$\nabla_{\mathbf{x}_i} p(X) = \nabla_{\mathbf{x}_i} U_i(\mathbf{x}_i), \quad \forall i \in N.$$

Moreover,  $\sum_{i \in N} v_i(\mathbf{x}_i)$  is concave in  $X$  (concave in  $\mathbf{x}_i$  by [Assumption 3](#) and zero in  $\mathbf{x}_j, \forall j \neq i$ ). Thus, it suffices to prove that the term

$$p_2 = - \sum_{t \in H} \left\{ \frac{(1 + \pi)c}{2} \cdot \left[ \left( \sum_{i \in N} x_i^t \right)^2 + \sum_{i \in N} [(x_i^t)^2] \right] + \frac{\gamma(n-2)}{2n} \cdot \sum_{i \in N} \left[ x_i^t \cdot \sum_{j \neq i} (x_j^t) \right] \right\}$$

is also concave, or equivalently that  $-p_2$  is convex. We have that  $\nabla_{\mathbf{x}_i} (-p_2) = \nabla_{\mathbf{x}_i} b_i(\mathbf{x}_i)$ . This yields

$$\nabla_{\mathbf{x}_i}^2 (-p_2) = \nabla_{\mathbf{x}_i}^2 b_i(\mathbf{x}_i) = \mathcal{J}_{c_i}^b,$$

which by [Lemma 1](#) is positive definite. Hence,  $p_2$  is also concave in  $X$ , as its Hessian matrix

$$\mathcal{H}^{p_2} = \nabla_X^2 p_2 = \text{blkdiag}(\{\mathcal{J}_{c_i}^b\}_{i=1}^n)$$

is a block diagonal positive definite matrix. Therefore,  $p$  is concave in  $X = (\mathbf{x}_1, \dots, \mathbf{x}_i, \dots, \mathbf{x}_n)$  as a sum of concave functions in  $X$ .

(iii) Since the potential function is concave and the players in the game seek to maximize it, it directly follows that best-response dynamics converges to the unique NE. ■

The second term of the sum at the right hand side of [Eq. \(12\)](#) introduces a price-discrimination component among users with different levels of flexibility. The ESP can control the magnitude of this discrimination by adjusting parameter  $\gamma$ , as will be analyzed in subsection *B* of this section. Thus, by increasing  $\gamma$ , users are increasingly incentivized to modify their consumption patterns. Note that  $\gamma$  does not increase the bills in general but only controls the way that the system's cost is shared among users. This provides useful intuition on the way [Eq. \(12\)](#) keeps the system budget balanced, and is proved formally below.

**Theorem 2.** *The billing rule  $b(\mathbf{x}_i)$ , given by [Eq. \(12\)](#), satisfies the budget balance property.*

**Proof.** It suffices to show that

$$\sum_{i \in N} b(\mathbf{x}_i) = (1 + \pi) \sum_{t \in H} \left[ C^t \left( \sum_{i \in N} x_i^t \right) \right].$$

By substituting  $b(\mathbf{x}_i)$  from [Eq. \(12\)](#), we have

$$\begin{aligned} \sum_{i \in N} b(\mathbf{x}_i) &= \sum_{i \in N} \left\{ (1 + \pi) \sum_{t \in H} \left[ \frac{x_i^t}{\sum_{j \in N} [x_j^t]} C^t \left( \sum_{j \in N} x_j^t \right) \right] \right\} \\ &+ \sum_{i \in N} \left\{ \gamma \cdot \left[ \sum_{t \in H} \left[ x_i^t \cdot \sum_{j \neq i} (x_j^t) \right] - \frac{\sum_{j \in N} \left[ \sum_{t \in H} (x_j^t \cdot \sum_{k \neq j} x_k^t) \right]}{n} \right] \right\} \end{aligned}$$

$$\begin{aligned} &= \sum_{t \in T} \sum_{i \in N} \left[ \frac{x_i^t}{\sum_{j \in N} [x_j^t]} \cdot (1 + \pi) C^t \left( \sum_{j \in N} x_j^t \right) \right] \\ &+ \gamma \sum_{i \in N} \sum_{t \in H} \left[ x_i^t \cdot \sum_{j \neq i} (x_j^t) \right] \\ &- \gamma \sum_{i \in N} \left[ \frac{\sum_{j \in N} \left[ \sum_{t \in H} (x_j^t \cdot \sum_{k \neq j} x_k^t) \right]}{n} \right] \\ &= (1 + \pi) \sum_{t \in T} \left[ C^t \left( \sum_{i \in N} x_i^t \right) \right] \\ &+ \gamma \left[ \sum_{i \in N} \sum_{t \in H} \left( x_i^t \cdot \sum_{j \neq i} (x_j^t) \right) - \sum_{j \in N} \sum_{t \in H} \left( x_j^t \cdot \sum_{k \neq j} (x_k^t) \right) \right] \\ &= (1 + \pi) \sum_{t \in T} \left[ C^t \left( \sum_{i \in N} x_i^t \right) \right] \end{aligned}$$

which completes the proof. ■

Furthermore, the following theorem verifies that [Requirement 2](#) holds for the proposed billing rule given by [Eq. \(12\)](#).

**Theorem 3.** *Game  $\Gamma$ , in equilibrium, satisfies the individual rationality property.*

**Proof.** The root vector  $\mathbf{x}_i^{\text{root}} \triangleq \{x_i^{\text{root}}\}, t \in H$ , for which  $b(\mathbf{x}_i^{\text{root}}) = 0$  can be found by solving [Eq. \(12\)](#):

$$\begin{aligned} x_i^{\text{root}} &= \frac{1}{2(1 + \pi)cn} \left[ - \sum_{j \neq i} (x_j^t) \cdot (n((1 + \pi)c + 1) - \gamma - 1) \right. \\ &\left. \pm \sqrt{\left[ \sum_{j \neq i} (x_j^t) \cdot (n((1 + \pi)c + 1) - \gamma - 1) \right]^2 + 4c\gamma n} \right]. \end{aligned}$$

Setting

$$\sum_{j \neq i} x_j^t \cdot (n((1 + \pi)c + 1) - \gamma - 1) = \alpha,$$

and

$$4(1 + \pi)c\gamma n = \beta,$$

we get

$$x_i^{\text{root}} = \frac{1}{2(1 + \pi)cn} \left[ -\alpha \pm \sqrt{\alpha^2 + \beta} \right],$$

which implies that there is always exactly one  $x_i^{\text{root}} \geq 0$ . By [Assumptions 1](#) and [2](#), we have  $v_i(x_i^{\text{root}}) \geq 0$ . Thus, from [Eq. \(5\)](#) we get that  $U_i(\mathbf{x}_i^{\text{root}}) \geq 0$ . This means that each user's utility is weakly positive, completing the proof. ■

## 6.2. The ESP's algorithm for constraint satisfaction

As discussed earlier, the ESP controls the system's cost via parameter  $\gamma$ . A low value of  $\gamma$  would lead to high energy cost, while a large value of  $\gamma$  would negatively impact the welfare. Since the ESP is agnostic of the users' valuation functions, determining the appropriate  $\gamma$  calls for a global optimization approach. We opt

**Table 1**

The proposed DSM procedure.

1	set $x_i^k = \bar{x}_i, \forall i \in N, k = 0, T_0, \gamma_0, \Delta_0$
2	<b>Repeat</b>
3	<b>Repeat</b>
4	for $i \in N$
5	user $i$ calculates best-response $\mathbf{x}_i(\gamma_k)$ from (11)
6	<b>Until</b> Nash Equilibrium
7	ESP calculates cost at NE ( $C_k$ )
8	$\Delta_k = (C_k - C_{ref})^2 - (C_{k-1} - C_{ref})^2$
9	$k = k + 1$
10	$T_k = T_0 \cdot 0.95^k$
11	ESP determines next $\gamma_k$ as a function of $T_k, \Delta_{k-1}, \gamma_{k-1}$
12	<b>Until</b> $\frac{\sum_{k=500}^k \Delta_k}{500} \leq \varepsilon$

for a Simulated Annealing (SA) method for determining  $\gamma$ . The SA method does not provide theoretical guarantees on the rate of convergence. In practice, however, this is not a real issue in our case, since we are only optimizing in one dimension. Also, as the ESP repeatedly performs this task, the search interval for  $\gamma$  can be drastically constricted by examining historical data. The entire DSM procedure is depicted in Table 1.

Parameter  $T_k$  is the so called ‘‘temperature’’ of the SA algorithm. In line 11, the SA algorithm determines the next value of  $\gamma_k$  based on a probabilistic calculation, which is not presented here due to space limitations.

## 7. Performance evaluation

In this section, we demonstrate the performance of the proposed architecture. In our simulation setup, we consider an optimization horizon of 24 hourly intervals,  $H = \{1, 2, \dots, 24\}$ , and  $n = 50$  users. The upper bound  $\bar{x}_i^t$  on user  $i$ 's consumption is chosen randomly with equal probability from the set  $\{1.5, 4, 5.5, 7.5\}$ . The feasible set  $h_i$  is modeled as a continuous set of timeslots  $h_i = [t_i^s, t_i^f]$  starting at timeslot  $t_i^s$  and ending at timeslot  $t_i^f$ . In order to model the afternoon peak demand, parameter  $t_i^s$  was picked from a uniform distribution in the interval  $[1, 17]$  for half the users, and in the interval  $[18, 21]$  for the other half. Parameter  $t_i^f$  was modeled as  $t_i^f = \min((t_i^s + d_i), 24)$ , where  $d_i$  was chosen randomly in the set  $\{3, 4, 5, 6, 7\}$ . Finally, the cost parameter  $c$  and the ESP's profit parameter  $\pi$  were set to  $c = 0.02$  and  $\pi = 0$ , respectively.

We tested the proposed architecture for three different user models (valuation functions) that satisfy Assumptions 1–3: Model A is taken from [1] and [39], model B from [4] and [37] and model C from [38]. Parameter  $\omega$  relates to the user's inelasticity/inflexibility in terms of his/her consumption.

**User model A:** The valuation function of user  $i$  is chosen as

$$v_i(\mathbf{x}_i) = \sum_{t=1}^H \left[ v_{max,i}^t - \omega_i \cdot (\bar{x}_i^t - x_i^t)^2 \right],$$

where  $v_{max,i}^t = \omega_i \cdot (\bar{x}_i^t)^2$ . This model captures the use case where a user's valuation function is temporally decoupled. In the experiments, parameter  $\omega_i$  was randomly selected in the interval  $[1, 2]$ .

**User model B:** The valuation function of user  $i$  is chosen as

$$v_i(\mathbf{x}_i) = \begin{cases} v_i^{max} - \omega_i \cdot \left( E_i - \sum_{t=1}^H x_i^t \right)^2, & \sum_{t=1}^H x_i^t < E_i \\ v_i^{max}, & \sum_{t=1}^H x_i^t \geq E_i \end{cases}$$

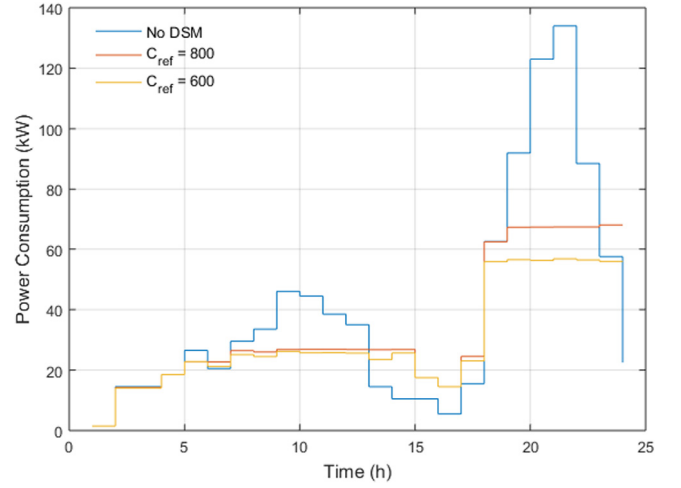


Fig. 1. Aggregated users' energy consumption curve – cost constraint.

Here,  $E_i$  is the user's desired energy in order to complete a task, which in the experiments was randomly chosen in the set  $\{4.5, 16, 22, 30\}$ . This model captures the use case where the user is interested only in his/her total consumption at the end of the day and not in the particular timeslots of consumption, which corresponds to completely shiftable loads. Parameter  $\omega_i$  was randomly selected in the interval  $[0.25, 1.25]$ , while we have set  $v_i^{max} = \omega_i \cdot (E_i)^2$ .

**User model C:** The valuation function of user  $i$  is chosen as

$$v_i(\mathbf{x}_i) = \begin{cases} v_i^{max} - \omega_i \cdot \left( E_i - \sum_{t=1}^H x_i^t \right)^2 \\ - \sum_{t=t_i^{des}+1}^{t_i^f} \left( \delta_i^{t-t_i^{des}} \cdot x_i^t \right), & \text{for } \sum_{t=1}^H x_i^t < E_i \\ v_i^{max} - \sum_{t=t_i^{des}+1}^{t_i^f} \left( \delta_i^{t-t_i^{des}} \cdot x_i^t \right), & \text{for } \sum_{t=1}^H x_i^t \geq E_i \end{cases}$$

where the last term expresses the user's discomfort from postponing consumption to later timeslots. Here,  $\delta$  is an elasticity/flexibility parameter related to the user discomfort due to shifted consumption, which in the experiments was randomly selected in the interval  $[1, 1.2]$ , and  $t_i^{des} = t_i^s + E_i/x_i^t$  is the desired timeslot for task completion. Naturally, we have  $t_i^{des} < t_i^f$ . Constant  $\omega_i$  is an elasticity/flexibility parameter related to the user discomfort due to reduced consumption, and was randomly selected in  $[0.25, 1.25]$ .

The results depicted in Fig. 1 were obtained assuming user model B. Fig. 1 shows the aggregated consumption of the users throughout the time horizon  $H$ , in the cases of (a) no DSM, (b) DSM with  $C_{ref} = 800$ \$ and (c) DSM with  $C_{ref} = 600$ \$. It is evident from the figure that the mechanism tacitly results in peak-shaving and valley-filling decisions in order to keep the system cost within the specified limit. Similar results were obtained (in Fig. 2) for the use case where the constraint is not posed on the system's cost, but instead a cap  $Y_{max}$  is posed on the aggregated consumption, so that  $\sum_{i \in N} x_i^t \leq Y_{max}, \forall t \in H$ .

In Figs. 3 and 4 we evaluate the Aggregated Users' Utility (AUU) performance of our scheme, defined as  $AUU = \sum_{i \in N} U_i$ , for all three user models. We depict the ratio of the AUU achieved by our proposed system over the  $AUU_{opt}$  that could be achieved by a central optimal solution if the users' valuations were known. In

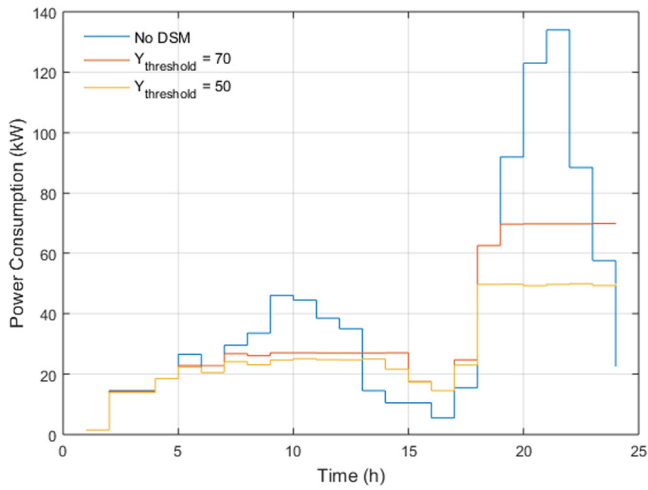


Fig. 2. Aggregated users' energy consumption curve – consumption constraint.

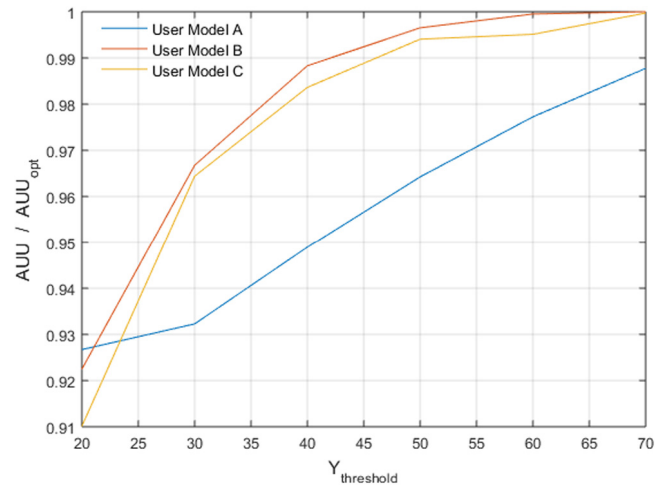


Fig. 4. Ratio between AUU and optimal AUU as a function of  $Y_{max}$ .

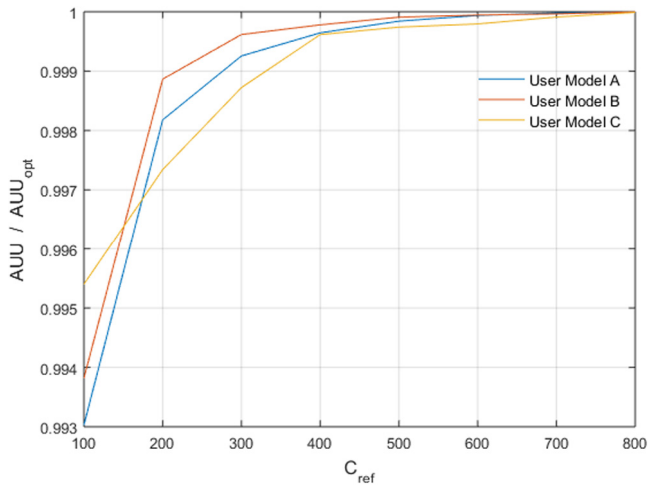


Fig. 3. Ratio between AUU and optimal AUU as a function of  $C_{ref}$ .

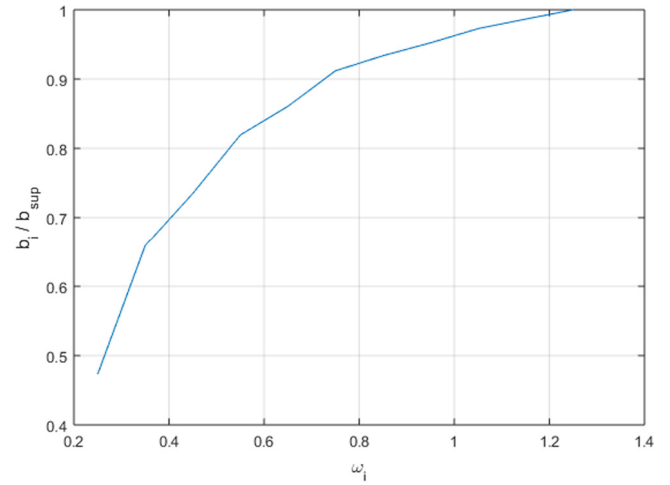


Fig. 5. Ratio between  $b_i$  and  $b_{sup}$  as a function of  $\omega_i$ .

particular, Fig. 3 depicts our scheme's  $AUU/AUU_{opt}$  performance as a function of the constraint  $C_{ref}$  on the total system cost, while Fig. 4 depicts the corresponding ratio for the use case where the constraint is not posed on the system's cost, but instead on the aggregated consumption  $Y_{max}$ .

In both Figs. 3 and 4 we observe that the  $AUU$  achieved by the proposed system reaches up to 97%–99% of that of the optimal solution. Even in extreme cases (for excessively low cap  $Y_{max}$ ),  $AUU$  is still more than 90% of the optimal solution.

A user's bill is affected by his/her inelasticity parameter  $\omega$ . In particular,  $b_i/b_{sup}$  expresses the ratio of  $i$ 's bill to his/her bill for the supremum  $\omega_i$ , denoted as  $b_{sup}$ . In Fig. 5 we show how the ratio  $b_i/b_{sup}$  is affected by  $i$ 's inelasticity  $\omega_i$ . The results were obtained under user Model B for  $C_{ref} = 600$ . We note that the user's bill is increasing with respect to his/her inelasticity parameter, so that inflexible users are penalized.

Finally, we present simulation results that relate to the computational time of the DSM procedure. The ESP communicates a value of  $\gamma$  to the end-users, who in turn participate in game  $\Gamma$ , calculating iteratively their best response until the Nash Equilibrium state is reached. Since the users solve their problems in parallel, each iteration of the game lasts as long as the 'slower' user needs to solve his/her (convex) optimization problem. The number of iterations, until the Nash equilibrium is reached, is depicted in Fig. 6 for up to 3000 users.

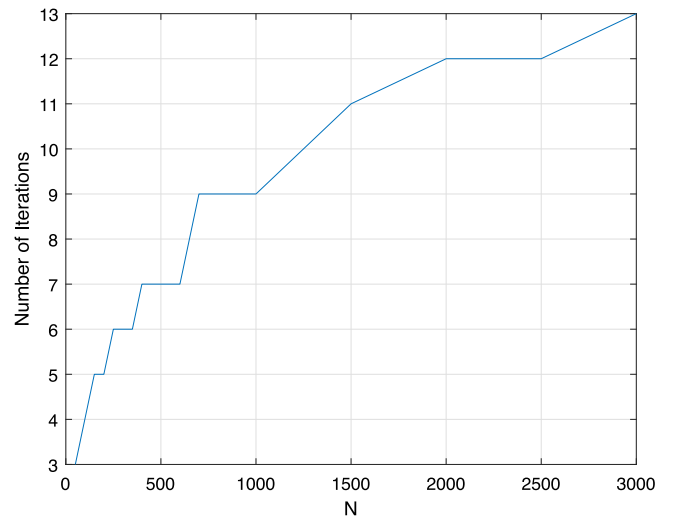


Fig. 6. Number of iterations until equilibrium as a function of the number of users.

Using an Intel Core i7 4 GHz, 64-bit, 16 GB RAM, computer, the convex optimization problem of each user can be solved at



around 0.1 s, which means that equilibrium for 3000 users is reached in less than 1.3 s. The users' (equilibrium) decisions are communicated to the ESP, and the ESP updates the value of  $\gamma$  if the target (cost or peak) has not been reached. The number of iterations of the SA algorithm does not depend on the number of users, but on the initialization of  $\gamma$  and the selection of algorithm's parameters. Studying historical data, the ESP should become better at warm-starting  $\gamma$ , and also can drastically compress the search space. On average, our SA algorithm does not take more than 200 iterations for an uninformed initial guess of  $\gamma$ , which means that the whole procedure can run approximately every 5 min in the worst case (200 iterations, times 1.3 s per iteration). If the ESP (examining historical data) is able to warm-start the search, then the number of iterations can be reduced to less than 30 and the procedure could run approximately every 39 s if need be.

## 8. Conclusions

We considered a game-theoretic formulation in order to model a system with price-anticipating energy consumers with coupled constraints. We presented a DSM architecture, along with a novel billing rule, that jointly achieve three generally desired properties (welfare maximization, budget-balance and controllability for constraint satisfaction). Moreover, the individual rationality property was preserved and the user's privacy was not compromised as opposed to the standard VCG approach.

These properties were proved theoretically but also verified through simulation experiments. Future research can extend the proposed mechanism to an online setting in order to account also for forecast errors.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## CRedit authorship contribution statement

**Georgios Tsaousoglou:** Conceptualization, Formal analysis, Methodology, Writing - original draft, Writing - review & editing. **Konstantinos Steriotis:** Conceptualization, Data curation, Validation, Visualization, Software. **Nikolaos Efthymiopoulos:** Conceptualization, Funding acquisition, Supervision. **Konstantinos Smpoukis:** Formal Analysis. **Emmanuel Varvarigos:** Funding acquisition, Supervision.

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