

# Fair Pricing Mechanism for Coalitions in Rural Areas

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**Abstract**—The constant expansion of Renewable Energy Source (RES) installations results in increasing the price of electricity in an unsustainable way due to the Feed-In-Tariff (FIT) policy currently being used. The challenge is to create a more liberalized market mechanism without, however, deterring further small scale RES investments, which remain costly. In the current paper, we consider the local electricity market in a given country or geographical area (rural, island, or other), where a certain portion of demand is asked to be covered from DERs. As DERs tend to be to some extent isolated from the main grid, it is possible and desirable for a group of DERs in a geographical area to be organized in a static local association that acts as a multi-plant organization. If the DERs (being small and many, thus “price takers”) negotiated as individual units with the market operator, they would achieve a price that is close to their marginal cost, which would be very small, much smaller than their average total cost. In this paper, we provide an algorithm that allows the coalition to offer its electricity production units at a higher price than normal market price without endangering their market share. The price they achieve is higher than if they participated in a complete liberalized market, but less than the tariff of the FIT policy. Interestingly, we find that the coalition has an optimal price at which its profits are maximized. Finally, we observed that if there are limited participants (coalitions) in the local electricity market, an upper bound needs to be set by a regulator, otherwise, the coalition will set the price at will.

**Keywords**— *Decentralized market, Rural area, Local Coalition, Fair Profit Distribution*

## I. INTRODUCTION

Feed-In-Tariff (FIT) policies have been used by many countries as a means to stimulate rapid investments on Renewable Energy Sources (RES) [1]. FIT is effectively a subsidy policy, where renewable energy generators are credited to sell their entire energy at higher prices than the normal market price. This, however, results in a total increase of the electricity price for the end consumer, especially if the policy succeeds and a large portion of energy is generated from RES. Thus, FIT is unsustainable and self-defeating, implying the need to rethink about the renewable energy market model.

However, direct participation of DERs to conventional liberalized electricity markets is not possible for many reasons. Firstly, conventional power plants sell their energy at a more competitive price than RES producers since they exploit their mature technology. That is, the average total cost for conventional power plants is much smaller than that of RES producers. Secondly, RES producers usually yield small-scale supplies making it difficult for them to directly participate in the electricity market. Thirdly, RES producers will be price takers in a liberalized market, as they will be numerous, very small and unable to affect the price. A price taker has to accept a price equal to its marginal cost which for RES producers is

very small, as they use no fuel, in contrast to the high average total cost (due to the cost of capital investments). As the marginal cost for RES producers is much smaller than their average total cost, they will be losing money and eventually get out of the market, while no new RES investments will take place in such a liberalized market. We argue in this paper that the virtual clustering of RES Microgrids into coalitions, called Virtual Microgrids (VMGs), which participate in the market as a single entity, will help them, together with related regulation, participate in the market as competitive monopolies that will have a stronger negotiating position in the market. Therefore, we need new tools able to allow the creation of energy associations (clusters) through which very small energy producers can confederate together to participate [2].

Several research works have been proposed to investigate the role of an aggregator in the electricity energy market. In [3] a decentralized architecture is presented, where a virtual aggregator (a software-based platform) allows DERs to trade electricity among them and the main market operator. In [4], a two stage market model is introduced for MGs power transactions. The proposed architecture includes an aggregator that acts as mediator between the MGs and the demands of the main power utility. In [5] an interesting approach for demand shaping is presented, where the utility gradually increases monetary compensation to aggregators, who compete in offering their demand shaping units. In [6], a game theoretic framework is introduced to form coalitions that minimize distribution losses. More specifically, the main idea of [6] is to minimize the power transfer between the coalition and the macro-station (MS). Then a parameter called “Shapley value” is used in order to fairly allocate the profits. The main limitation of all the aforementioned approaches is that they examine the market from the buyers’ perspective (utility or broker) far from real decentralized markets

In this paper, we provide a model for the RES electricity market in a given country or a given geographical area (e.g., rural area or island) so that the local DERs receive the highest possible profits without, however, going against the principles of a liberalized market mechanism. The price at which RES energy is sold is a parameter that can be adjusted to some extent at will, according to some specific logic and criteria that can be enforced by a regulator. This price will no longer have to be equal to the (almost zero) marginal cost of RES producers, as it would have to be if many small producers negotiated on their own in a liberalized market. In particular, the price can be adjusted so that it is set equal to the average production cost of RES producers (so that they don’t go out of business) or even larger than that (so that RES investments are encouraged), even though this requires appropriate regulation, as will be described in Section II. The justification for setting a price higher than the liberalized market price is that RES

produced electricity is qualitatively very different than that produced by other (fossil energy) sources, because of the environmental, geographic and other positive externalities that come with it.

For the special case of a rural (or island) area, the power network in rural areas can either be completely isolated from the main grid or it can be connected through a lengthy power line. In the first case, the DERs in the rural (or island) area are almost “obliged” to act cooperatively, otherwise the delivery of electricity cannot be secured. The second case, which we closely examine in this paper, implies a decentralized market which is not completely unaffected by other major big energy producers that supply the market. As already implied, due to geographical isolation, we naturally, assume that some of the DERs in the neighborhood are united in a coalition and act in the market as a single entity. Then, by exploiting their cooperation, we propose an algorithm that allows the coalition (represented by an aggregator) to achieve a more profitable bid than a completely liberalized market. In particular, through an iterative process the aggregator manages to find the price that maximizes coalition’s profits and by slightly controlling their output, DERs can achieve higher profits. For the profit distribution among the coalition we used weighted max-min algorithm [9] in order to distribute the profits in a fair way.

## II. PRICE TAKERS VERSUS COMPETITIVE MONOPOLIES

There are at least 3 ways in which the small producers can compete in a liberalized market:

- As individuals and without help from regulatory authorities. Then the RES producers, being small and numerous, will have to be “price takers”. The price they obtain in the market is equal to their Marginal Cost (MC). This price corresponds to point 1 in Fig. 1a, and may be very low and unsustainable (no new RES investments and existing RES producers will eventually go out of business).
- As member of a cluster (VMG) without regulatory help. In that case each VMG can be viewed as a competitive monopoly. A VMG can set its own price, but it is competing against other competitive monopolies (fossil fuel electricity producers). The VMG will ask for a price that exceeds its average total cost (ATC, which takes into account initial investment and includes a small profit), and the price it will be able to get will be the one determined by the rest of the market players (which determined the Demand curve for the RES energy produced by the VMG). This price corresponds to point 2 of Fig. 1a.
- As member of a cluster (VMG) but with regulation helping change the Demand. The regulation in this case will force the Demand curve to go up. The price the VMGs get is point 3 in Fig. 1b, which regulation can adjust (by adjusting the Demand curve) so that it is above the ATC of the cluster. If the price obtained is indeed above the ATC, the VMG as a whole makes a profit and new investments in RES energy will take place in the long run.

An interesting special case, which we investigate in this paper, is the case where the Regulator intervenes in the regional (or country) market, by asking that a certain amount  $D$  of electricity in the region (e.g., 20% of the total electricity consumed in the region) must be produced by RES Microgrids.

In that case, the Demand curve is flat (inelastic), with the RES quantity supplied being independent of its price. It is the only the VMGs or the individual RES MGs that can compete for that energy  $D$ . A VMG has the advantage that it will use in its negotiations its ATC instead of its MC in setting its price, as it is no longer a price taker but a price maker.

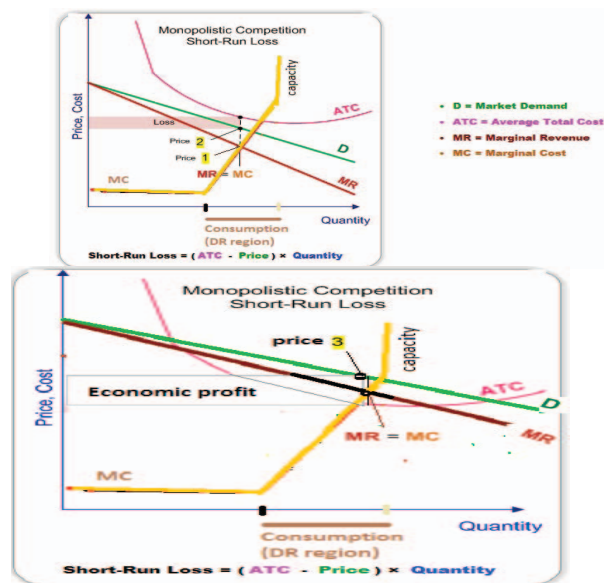


Fig. 1: (a) The case of Monopolistic (VMG) competition without regulatory help. The prosumer will set its quantity at the point where Marginal Revenue (MR)=Marginal Cost (MC). If the prosumer is a “price taker” (an individual MG, the price it gets corresponds to point 1, i.e., it is equal to its MC since the prosumer has no negotiating power. If, however, the prosumer is a VMG competitive monopoly, it gets the price 2 determined by its completion with other VMGs (and possibly the rest of the market, if RES energy is not treated as a separate type of energy). (b) Regulation can help raise the demand curve  $D$  so that the price obtained for RES electricity is higher. If the price obtained is above ATC, then the VMG makes a profit.

In Fig.1 we note that the MC of a RES producer may be very small (close to 0 in practice, as the “fuel” is for free), and it is much smaller than the ATC of the producer. It is important to note that in the figure we assume that each MG is actually a prosumer (i.e., both a producer and a consumer), which is a usual case. The MG has a certain capacity and part of its production is locally consumed. The prosumer can use Demand Response (DR) in order to increase the amount of electricity it can make available for sale to the grid, but this happens with an increasing MC (modelling the increasing dissatisfaction of the prosumer when curtailing its own consumption). If the RES MG is only a producer and has no storage, then the DR region and the corresponding part of the MC curve would be absent.

## III. ENVIRONMENTAL AND GEOGRAPHICAL EXTERNALITIES OF RES PRODUCTION

We consider a system (country or geographical area) consisting of MGs that produce (and also probably consume) electricity based on renewable energy sources (RES). The electricity produced by a RES is qualitatively very different than that produced by other (fossil fuel energy) producers, because of the environmental and other benefits that come with it.

The environmental benefits translate, in the economic domain, into the avoidance of steep CO<sub>2</sub> monetary penalties

that the rest of the market players (e.g., the main energy producer) would have to pay, unless a certain percentage of the energy production comes from RES. Each unit of RES energy can therefore be seen as being accompanied by a “green credit” that benefits the market at which the RES energy is produced and consumed. If the RES energy was sold in the usual energy market (at “normal” market prices), these green credits that come from RES production would benefit the entire market (especially the main producer) and not the RES producers/MGs that generated these benefits. In other words, these green credits constitute a positive externality<sup>1</sup> that is due to the RES producers but mainly benefits the other unrelated market players. Another positive externality that comes from MG production is that such energy production is usually close to the consumers, and thus avoids the a) transportation losses and the b) big investments needed for improving the distribution network so as to transport the energy over large distances. If the RES energy was sold in the usual energy market (at “normal” market prices), this positive externality would again benefit the other market players (e.g., the DSO) and the RES producers would get nothing out of it.

Our premise is that these two important positive externalities should be used to benefit those that create them, that is, the RES producers themselves, and should not be captured and exploited for free by the other market players. There are at least three ways in which the green and the geographic advantages (positive externalities) of RES production will be returned to benefit the RES producers:

- a) The use of feed-in tariff policies. This is the current practice at many EU and other countries, but is inflexible (e.g., difficult to adapt in time based on supply and demand), and it is difficult to calculate and justify economically
- b) The use of geographic location dependent pricing (for the geographic externality) and the pricing of the green credits together with energy itself (for the green externality)
- c) The formation of a special market for RES energy, where these positive externalities can be returned to RES producers, by settling an above market price (we will sometimes call it “unfair”) for the RES participants. This is equivalent to the Regulator establishing a Demand curve  $D$  for RES electricity and probably for each specific area (e.g, rural or island) through its own intervention.

The approach (c) is the one examined in the present paper. A KWh produced by a RES that is also close to the consumer is worth considerably more than a KWh produced by a fossil energy power plant that is located far from the consumer. Thus a MG and especially a RES Kwh is economically distinguishable from a usual (fossil energy power plant) Kwh, and a separate market, (through the Demand curve  $D$ ), has to be created for them.

#### IV. SYSTEM MODEL

##### A. Decentralized Rural Market

Important environmental concerns (Greenhouse effect), force the electricity market to operate under certain regulation

<sup>1</sup> An externality is a consequence of an economic activity that is experienced by unrelated third parties. An externality can be either positive or negative.

rules that promote the usages of renewable energy sources. Under this consideration, EU obliges its countries to include at least 20% from RES within their entire energy consumption, or otherwise be subject to high penalties [7]. In another EU regulation directive that drives RES market, power companies (or utilities) that use obsolete generators are also penalized for CO<sub>2</sub> emissions. Thus, it is evident that significant demand for RE comes from regulatory constraints that are hard for power companies to avoid. In the near future, demand will also come from environmentally sensitive consumers who will be willing to pay a larger price for electricity produced by RES.

In general, there is no universally applicable description of the rural and island electricity market. For example, in Greece many small islands and some remote areas are difficult to connect them with the main grid as it is extremely costly. In these locations, the electricity distribution is often erratic leading to outages and other low quality services. More frequently, however, in areas with dense and concentrated populations, extension of the grid may be feasible and cost-effective. In any case, the ease of communication among the DERs makes it possibility for them to unite through ICT technologies in a cooperative manner and form a coalition (e.g., the DERs in a given geographical area, or DERs that share some other characteristic or relationship, including similarity or dissimilarity in their production patterns, etc). In this paper, we assume that some DERs in a given geographical area are organized virtually in a coalition through a software platform and act as a single entity. We also assume that the area is connected with the main grid allowing other MGs/DERs to participate in the local RES market. The decentralization of the market helps to address the demand locally so as to minimize the power transportation losses, reduce the need for additional investments in the transportation network, and simplify the market operation.

In what follows we consider a local market operator (MO) or a local utility that is operating in a given area  $l$  where a total  $M$  of DERs (generating RES) are also active. We assume that the MO or utility is forced (by regulation or other reasons) to buy an amount  $D(l)$  of electricity from the RES producers in area  $l$  in order to satisfy a corresponding local RES demand :

$$\sum_{i=1}^M x_i = D(l) \quad (1)$$

where  $x_i$  is the electricity units supplied by the  $i$ -th DER.

Local RES electricity market is assumed to operate in a manner similar to a typical liberalized electricity market. Some RES producers may choose to belong to a coalition (VMG) that acts as competitive monopoly, while the remaining RES producers may act as individuals. More elaborately, an energy pool is assumed to aggregate the supply bids during negotiation phase. At its expiration, the local RES MO broadcasts the accepted bids of RES supply and the clearing price of the market. A VMG in the local market as well as any MGs/DERs that are outside this coalition act in a competitive manner.

##### V. SIMILARITY TO OPEC MODEL

Our proposed RES market model presents similarities with the oil market model. As is well known, the Organization of the Petroleum Exporting Countries (OPEC) aims at achieving higher prices for its members than those normally achieved in a

free market, by deciding the total volume to be produced by its members, and setting a target price for petroleum. The target price is above the price that would normally prevail in the market, given the low marginal production cost of some of its members. Then OPEC allocates production quantities to its members and makes sure that no OPEC member exceeds that (“cheats”). Without this allocation mechanism enforced, the production of the OPEC members would be much larger and so would be the total supply, driving prices down.

This way, instead of the price being determined from the production costs and the users’ demand, some big players (actually, associations of players) limit the production to affect the market price at higher levels. As more distributed RES producers enter the market, the normal market price would decrease for a given demand, providing no incentives for more players to participate in. One way to address this is for “big energy production associations” to agree to limit their production in order to adjust market price at higher values. For this policy to be effective, the increase over the market price should be such that both small and big energy producers benefit, i.e., their revenues, calculated as the product of price by allocated production amount, are greater than the respective revenues obtained from traditional market. This means that the proposed algorithm should yield benefits to both small and big RES Microgrids over open market and, simultaneously, provide incentives for RES investments. The justification for the higher prices targeted by RES producers are the aforementioned positive externalities that they create.

Similarly with the OPEC model for petroleum production, a VMG is a coalition of RES producers. There are RES producers who may opt not to participate in the VMG. OPEC is also a competitive monopoly in the sense that petroleum competes against other fossil fuels (like coal, gas, etc) and renewable energy.

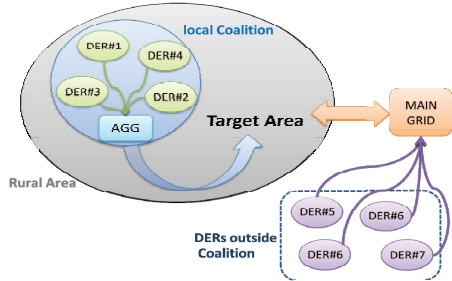


Fig. 2: System model for a rural or island electricity market

### A. Production Cost Modeling for RES DERs

Generally, the energy cost consists of two main factors; the energy production cost  $G_i(x)$ , and the transportation cost  $T_i(x, l)$ . Function  $T_i(x, l)$  models the energy losses caused for delivering the  $x$  energy units from the physical location of the  $i$ -th DER to the region of demand  $l$ . Since RES production requires no fuel, the marginal cost of producing a unit of energy is limited to the almost fixed operational and maintenance costs that the investment needs. However, since we assume prosumer Microgrids (both producers and consumers), we will assume in what follows that the production cost increases linearly with the output:

$$G_i(x) = \begin{cases} \gamma_i \cdot x, & 0 \leq x \leq X_{i,\max} \\ \infty, & \text{otherwise} \end{cases} \quad (2)$$

where  $\gamma_i$  is a parameter modelling the MC of producing an additional unit of electricity.  $X_{i,\max}$  is the maximum producing capacity of the plant ( $Kwh$  produced in time unit). In fact in our experiments  $\gamma_i$  will be taken to be a function of time, and equal at any time instant to the price at which electricity is sold to the MG by the grid when the MG is a consumer.

$T_i(x, l)$  can be decomposed into (i) the power losses over the line, which are of quadratic form:  $R_i(l)/V_0^2 \cdot x^2$ , where  $V_0$ ,  $R_i(l)$  the voltage and the resistance of the power line, respectively for the  $i$ -th DER to distribute the  $x$  units to region  $l$  and (ii) the cost of transforming the energy from low voltage to high voltage and vice versa, [8]. That is,

$$T_i(x, l) = \gamma_i \cdot \left( \frac{R_i(l)}{V_0^2} \cdot x^2 + \delta_i \cdot x \right) \quad (3)$$

where  $\delta_i \cdot x$  models the transformation cost and  $\gamma_i$  is the price of electricity as indicated in (1). Eq.(3) implies that the cost of the  $i$ -th MG is dependent on its distance from the target region. Note that, we get for the maximum quantity that can be offered in region  $l$ :

$$X'_{i,\max}(l) = \frac{V_0^2 \cdot \left( -1 - \delta_i + \sqrt{(1 + \delta_i)^2 + 4 \frac{R_i(l)}{V_0^2} X_{i,\max}} \right)}{2 \cdot R_i(l)} \quad (4)$$

This means that the maximum supply a DER can deliver, varies depending on the region. Unifying the above costs we obtain that the total cost of  $i$ -th producer for supplying  $x$  units to region  $l$  is of quadratic form:

$$C_i(x, l) = G_i(x) + T_i(x, l) = \begin{cases} a_i x + b_i(l) x^2, & 0 \leq x \leq X'_{i,\max}(l) \\ \infty, & \text{otherwise} \end{cases} \quad (5)$$

where  $a_i = \gamma_i(1 + \delta_i)$ ,  $b_i(l) = \gamma_i \frac{R_i(l)}{V_0^2}$

In the following, we omit for simplicity purposes the variable  $l$  denoting the area, since we always refer to a specific location.

## VI. PROBLEM FORMULATION

### A. Profit maximization of DERs who negotiate individually, as “price takers”

Supposing that  $\lambda_i$  is the price bid offered to the  $i$ -th DER, the DER tries to maximize its profit revenue by solving the following equation:

$$x_i^* = \arg \max_x \{ \lambda_i \cdot x_i - C_i(x_i) \}, \text{ s.t. } 0 \leq x_i \leq X'_{i,\max} \quad (6)$$

Differentiating (6), we have that the (unconstrained) optimal supply of  $i$ -th DER at price  $\lambda_i$  is:

$$x_i = (\lambda_i - a_i) / (2 \cdot b_i) \quad (7)$$

This expression is, however, misleading for the case of many small DERs, since the capacity constraints also have to be satisfied. In other words, the supply given by Eq.(7) is optimal for the  $i$ -th DER only if it falls in the interval  $[0, X'_{i,\max}]$ . This is typically *not* the case for the value given by (7), since  $b_i$  takes a very small value and the unconstrained optimal supply exceeds and has to be curtailed to  $X'_{i,\max}$ . This means that as long as the price  $\lambda$  offered by the market slightly exceeds  $a_i$  (which is the MC) the prosumer will be willing to sell its entire capacity to the market (that is why he is called a price taker).

### B. Profit maximization of a set of DERs who negotiate jointly as a non-monopolistic VMG

We now consider the case where the  $N$  DERs that are active in a given area unite in a group (a VMG) that negotiates as a single entity in the market, the situation changes and the DERs are no longer price takers. The target of the group is of course to maximize the total profit of the VMG. Supposing that  $\lambda$  is the price offered to the VMG and given the constraint about the local demand  $D$  that the MO must meet, the formulation of the profit maximization problem for the VMG is the following:

$$\begin{aligned} & \max\{\lambda \cdot \sum_{i=1}^N x_i - \sum_{i=1}^N C_i(x_i)\}, \\ & \text{s. t. } \sum_{i=1}^N x_i = D, \quad 0 \leq x_i \leq X'_{i,\max} \end{aligned} \quad (8)$$

To be most precise, and since we have to take into account the capacity constraint, the optimization of Eq. (8) must be performed through the application of an iterative (and distributed) process. In this case, the aggregator iteratively reduces the common price of the association in order to satisfy the constraint  $\sum_{i \in S} x_i = D$ .

$$\lambda(k+1) = \lambda(k) + \varphi [D - \sum_{i \in S} x_i] \quad (9)$$

where  $k$  is the iteration index and  $\varphi$  a constant variable denoting the reduction step of the algorithm. For a given price  $\lambda(k)$ , the VMG performs the optimization of Eq. (8) in order to obtain the new supplies  $x_i(k)$  of  $i$ -th DER,  $i=1,2,\dots,N$ .

The control of the output sold,  $x_i$ , is not directly performed by switching on or off the PV panels; rather the DER chooses either to consume the additional units onsite, or to store them (e.g., in order to generate more profits by offering those units at time frames that are more favorably priced).

Note that the price  $\lambda$  at which the DERs of the VMG sell their produced electricity is common for all the DERs. Therefore, all the DERs gain when the MO is forced to increase its price in order to get more supply and meet the required demand  $D$ . The result would be same if the DERs were negotiating on their own while learning the prices offered to the other DERs, increasing their price when another DER was offered a better price (thus, they are not price takers).

### C. Profit maximization of a VMG that acts as a competitive monopoly

In this case, the DERs form a VMG coalition that offers its product to the market as a competitive monopoly. By competitive monopoly we mean that the product it offers is diversified from the products offered by other market participants but it is not a strict monopoly as it competes against them. More specifically, the product offered by the VMG is "local green electricity" and the products offered by the other participants are "electricity produced by fossil fuel" or "non-locally produced electricity". The product "electricity produced by fossil fuel" is differentiated from that produced by the RES VMG because it may involve additional costs in the form of CO2 penalties and the product "non-locally produced electricity" is distinguished from that produced by the VMG competitive monopoly because it involves transportation losses and investment costs for improving the transportation grid. As explained in Section II, the Regulator may increase the Demand curve for the particular "local green electricity"

product of the VMG competitive monopoly, and could even make it a pure monopoly if it wished (e.g., by making "dirty energy" illegal).

To the degree that the externalities can be monetized or the degree to which RES is helped by the Regulator, the coalition (VMG) can offer its units at a higher price than it would in an open market so as to increase its profits. As already stated, the justification for offering the RES units at this higher price are exactly the positive externalities that the RES production offer and are (in the absence of a higher price) mainly enjoyed by unrelated players of the market. Achieving of this increased price implies that the coalition has a tight control over the production output of its members. The additional units of the coalition that might not be supplied in the market can either be stored or can be consumed onsite. Storing the additional units provides a twofold opportunity: a) the coalition can bid at another timeframe where the aggregated production pattern of coalition is small, or b) in order to successfully face an emergency event, for example where the agreed with the MO supply cannot be met. Of course the price set should be lower than the price that the rest players of the market are willing to supply this distant region.

Suppose that the set of DERs that form the VMG coalition is denoted with  $S$ , the complementary set (DERs outside the coalition) is denoted with  $S'$ ,  $\lambda_c$  is the price set by the coalition,  $X_S = \sum_{i \in S} x_i$  is the coalition's supply units,  $C(X_S)$  is the total cost of the coalition for supplying  $X_S$  units and  $X_{S'}(\lambda_c) = \sum_{j \in S'} x_j(\lambda_c)$  is the supply of group in  $S'$  for market price equal to  $\lambda_c$ , then the coalition's goal is to maximize its aggregated profits  $\Pi_c$  for the residual demand:

$$\begin{aligned} & \max_{\lambda_c, X_S} \{\Pi_c = \lambda_c \cdot X_S - C(X_S, D)\}, \\ & \text{s. t. } X_S = \sum_{i \in S} x_i = D - X_{S'}(\lambda_c), \quad 0 \leq x_i \leq X_{i,\max} \end{aligned} \quad (10)$$

The price  $\lambda_c$  that the coalition can set must be obviously less than the price  $\lambda_0$  that the cluster  $S'$  is willing to offer the requested amount  $D$ , otherwise the coalition would have nothing to gain. Since the DERs of  $S'$  participate in a competitive environment, and assuming that negotiation is done with disclosure of the price regarding the bids made to other DERs (case B of Section V), we have that price  $\lambda_0$  is:

$$\begin{aligned} & \lambda_0 = \arg \max_{\lambda} \{\lambda \cdot \sum_{j \in S'} x_j - \sum_{j \in S'} C_j(x_j)\}, \\ & \text{s. t. } \sum_{j \in S'} x_j = D, \quad 0 \leq x_j \leq X_{j,\max} \end{aligned} \quad (11)$$

Obviously, the VMG aggregator can set the price to be  $\lambda_c = \tau \cdot \lambda_0$  with  $\tau \in (0,1)$ .

## VII. THE VMG COALITION'S BIDDING STRATEGY

### A. Maximizing the VMG coalition's profits

In order for the coalition to offer its units at a higher price (than would normally give), the aggregator must have complete information about the market. Then, the coalition can solve Eq. (11) to obtain the market price  $\lambda_0$  for  $D$  units by the group  $S'$ . Let us denote as  $X_{S'}(\lambda) = \sum_{j \in S'} x_j(\lambda)$  the supply of set  $S'$  for market price equal to  $\lambda$ . Then, if the VMG coalition sets the price at  $\lambda = \tau \cdot \lambda_0$  the aggregator knows that the DERs outside the coalition would like to supply the market with a total of  $X_{S'}(\tau \lambda_0)$  units. Obviously, the constraint of Eq.(10) becomes:

$$X_S(\tau \cdot \lambda_0) = D - X_{S'}(\tau \cdot \lambda_0) \quad (12)$$

and Eq.(10) can be rewritten as:

$$\max_{\tau, \lambda_0} \{(\tau \cdot \lambda_0) \cdot X_S - C(X_S)\}, \text{ s.t. } X_S = D - X_{S'}(\lambda_c) \quad (13)$$

The  $X_S$ ,  $C(\cdot)$  of Eq.(13) define the behavior of the coalition in determining of coalition's supply and cost function. Since the coalition acts as a multiplant organization, aggregator should instruct its members to provide in the market their cheapest units in order to minimize its total cost.

$$C(X_S) = \min\{\sum_{i \in S} C_i(x_i)\}, \text{ s.t. } \sum_{i \in S} x_i = X_S \quad (14)$$

Then the goal of the aggregator can be rewritten as:

$$\begin{aligned} & \max\{\lambda \cdot \sum_{i \in S} x_i - \sum_{i \in S} C_i(x_i)\}, \\ & \text{s.t. } \sum_{i \in S} x_i = D - X_{S'}(\lambda) \end{aligned} \quad 99(15)$$

which can be solved in a distributed way [see DFA].

In this way the VMGs profits are maximized (also accounting for transportation losses). However, a fair sharing strategy should be obtained in order to distribute the profits among the VMG members, otherwise some DERs from the association might cease their cooperation with the aggregator.

Relaxing the capacity constraints for  $X_{S'}$  and  $X_S$  (i.e.  $0 \leq x_i \leq X_{i,max}$ ,  $0 \leq x_j \leq X_{j,max}$ ) and including Eq.(14), then Eq.(13) could be solved analytically using Lagrange Multiplier in order to acquire the unconstrained optimal value  $\tau^*$ :

$$\tau^* = \frac{2 \cdot D + \sum_{i \in S} \frac{a_i}{b_i} + \sum_{j \in S'} \frac{a_j}{b_j}}{\lambda_0 \cdot \left( \sum_{j \in S'} \frac{1}{b_j} + \sum_{i \in S} \frac{1}{b_i} \right)} \quad (16)$$

Unfortunately, the value of Eq. (16) may correspond to exceeding some capacity constraints, and since the constraint of Eq. (13) changes at every iteration in a nonlinear manner, an iterative algorithm needs to be implemented in order to acquire the optimal value for  $\tau$ . While Eq.(16) cannot be used directly however it shows that there is an optimal value for  $\tau$  that maximizes profits of coalition. In Table 1 follows the Best Price Algorithm (BPA), we estimate the optimal value  $\tau$ .

Table 1: The Best Price Algorithm (BPA)

1. Calculate the price  $\lambda_0$  that the other DERs supply the entire demand  $D$  to the region  $l$  [see eq.(11)].
2. Calculate the price  $\lambda_{open}$  that the DERs of all the market ( $S \cup S'$ ) supply the entire demand  $D$  to the region  $l$ . (i.e. Eq.(7) for all DERs in the market).
3. Set  $k=0$  and the price at  $\lambda_c(k) = \lambda_{open}$
4. While profits are not decreased
  - 4.1. Calculate the corresponding supplies of group  $S'$ ,  $X_{S'}(\lambda_c)$ , as obtained from optimization of (7).
  - 4.2. Ask from the coalition's members to offer the  $X_S=D-X_{S'}(\lambda_c)$  cheapest units to the market using the DFA.
  - 4.3. Calculate total profits of coalition.
  - 4.4. If profits of previous step are decreased, then stop  
Else, increase  $k=k+1$  and go to step 4.

The BPA apart from offering the optimal price to the aggregator, it also defines the total units that the coalition needs to offer in the market at this price.

### B. Fair Distribution of Profits

In order to fairly distribute the profits  $\Pi_c$  among the coalition's members, we make use of *weighted max-min* algorithm. This algorithm is generally used in order to share a

limited resource among multiple users [9]. Let us clarify the algorithm's process using an example. In particular, let us assume that a total of 24 units need to be allocated among four users that have different priorities (weights) and that have capacities [10,3,5,15] units respectively. Obviously, some of the users will receive less units due to insufficient resources. Moreover, let us assume that first and fourth user have weight of 2 and that the other two have weight 1. The amount of weight determines the priority of each user and the bigger the weight the higher the priority. At the first step, this algorithm divides the 24 units into  $2+1+1+2=6$  equal parts (note that the sum of weights determines the divider):  $24/6=4$  units and then allocates them among the users depending on their weights. In particular, the first user receives  $2 \cdot 4=8$  units. The second needs less than  $1 \cdot 4=4$  units and, thus, he receives 3 units leaving 1 unit unused. The third receives  $1 \cdot 4=4$  units and the last  $2 \cdot 4=8$  units. After this procedure 1 unit is left unused and is again divided into equal  $2+0+1+2=5$  parts, since second user has covered his demand:  $1/5=0.2$  units. Similar, to previous step: first and last receive  $2 \cdot 0.2=0.4$  units, second receives nothing and third receives  $1 \cdot 0.2=0.2$  units. Total allocation of resource among the four users is: [8.4/10, 3/3, 4.2/5, 8.4/15]. In Table 2 follows the weighted max-min algorithm applied in our case:

Table 2: The weighted max-min algorithm (WMM)

1. Define demand  $D$  to be allocated, weights  $w_i$  and maximum capacities  $X_i$  of each user
2. Set residual demand  $rd=D$ , set allocated resources of each user  $y_i=0$
3. Until  $rd \leq 0$ 
  - 3.1. Define portion of demand to be allocated:  $pd = d/\text{sum}(w_i)$
  - 3.2. To each user that has  $y_i < X_i$ :
    - 3.2.1 If  $y_i + pd < X_i$  set  $r = pd$  and set  $y_i = y_i + pd$   
Else set  $r = X_i - y_i$  and set  $y_i = X_i$ .
    - 3.3. Update residual demand  $rd = rd - r$

The definition of weights determines the distribution of profits among the coalition's members. In this paper, we allocate the profits in a "production-based fairness" manner, by measuring the contribution of each member of the coalition to the total supply for price  $\lambda_c = \tau \cdot \lambda_0$ , should none of them be in the coalition. The reason for adopting this notion of fairness is that the most effective DERs lose more by participating in the coalition (since they need to constraint more production units). Thus, the compensation should be generous to them, in order to provide incentives for entering in the coalition. That is, weights are determined by:

$$w_i = \frac{x_i(\lambda_c)}{\sum_{i \in S} x_i(\lambda_c)} \quad (17)$$

where  $x_i(\lambda_c)$  is the normal supply of  $i$ -th DER at price  $\lambda_c$  as given from (6).

### C. The Coalition's Bidding Algorithm

The Coalition's Biding algorithm (CBA) that the local aggregator uses to determine the optimum price to be set in order to maximize its profits is following in Table 3.

Table 3: The Coalition's Biding Algorithm (CBA)

1. Calculate the price  $\lambda_0$  that the other DERs supply the entire demand  $D$  to the region  $l$ .
2. Find the optimal price  $\lambda_c^*$  and optimal supply of coalition  $X_S^*$  using BPA.
3. Obtain the cheapest units of its members that satisfy  $X_S^*$ .

4. Obtain the supplies for market price  $\lambda_c^*$  of DERs that belong in coalition ( $x_i(\lambda_c^*)$ ).
5. Obtain the weights as defined by Eq.(17) and distribute fairly the profits  $I_c$  among the coalition's members using the fair scheduling algorithm [see WMM].

### VIII. SIMULATION RESULTS

We use three metrics to assess the gains of the proposed algorithm. Metric  $m_u$  represents the price at which power utility buys the requested amount  $D$ . Obviously, the less the price the bigger the benefit for the power utility. Metric  $m_c$  defines the average profits of all DERs in the market. Obviously, the higher the value of  $m_c$ , the better from the DERs' perspective. Metric  $m_w(S_w)$  derives the average profits of the DERs that belong into the set  $S_w$ . The higher its value the more beneficial it is for the DERs to participate in the set.

$$m_u = \lambda_c \quad (18)$$

$$m_c = \frac{1}{M} \cdot (\sum_{i=1}^M \lambda_c \cdot x_i - \sum_{i=1}^M C_i(x_i)) \quad (19)$$

$$m_w(S_w) = \frac{1}{|S_w|} \cdot (\sum_{i \in S_w} \lambda_c \cdot x_i - \sum_{i \in S_w} C_i(x_i)) \quad (20)$$

where  $\lambda_c$  is the market price,  $M$  stands for the number of DERs in the market,  $S_w$  is the set of the DERs that we are interested in,  $x_i$ ,  $C_i$  are the supply and the cost of  $i$ -th DER respectively.

#### A. System Model

In our simulations, we set up a distribution network within a square of  $150 \times 150 \text{ km}^2$  centered at the target area (the point where the demand is headed for) and the DERs were randomly deployed within this area. The resistance between any two nodes (DERs) and the voltage line (which define  $b_i$  parameters) are set to  $R_0=0.2$  ohm per km and 230V, respectively, which are practical values in the lower level of the distribution network [8]. For simplicity, we assumed that  $\delta_i=0$ . Typical small scale installations have capacities within the range [1kW-30kW]; thus, we chose the capacities of the DERs as independent Gaussian random variables with mean 15kW. In order to evaluate realistically a day-ahead market, statistical data from Sardinia's and Greek electricity market were used. In particular, production output of wind installations within Sardinia on 23/02/2015 were used for the definition of the hourly production pattern of the DERs [10]. Similarly, demand of Sardinia on the same day was used in order to define the hourly requested demand pattern [10]. For the definition of generation cost functions (i.e., the  $a_i$  values) we used realistic consumption patterns of households using dataset of [11]. Then, the  $a_i$  parameters were matched to the corresponding electricity prices of [12]. Due to computational limitations we used 1000 DERs able to cover a smaller portion of the actual demand. Nevertheless, without any loss of the validity of our results we used the above daily demand pattern in the range [2-4] MW. Finally, we assumed that the local coalition consisted of the closest DERs in the target area. All results are averaged over a large number of runs with different random positions, capacities and  $a_i$ ,  $b_i$  parameters for the DERs.

#### B. Results

In Figs.3-6 we depict the performance of the metrics defined in previous subsection with respect to the proposed algorithm. In the above figures " $COAL_S=x$ " represents the

proposed algorithm for coalitions and  $x$  is the share of the total production that the coalition has in the market. For instance,  $COAL_S=0.3$  means that the coalition obtains 30% of total production of the DERs in the market. "OPEN" is the market with no coalition (i.e.  $COAL_S=0$ ), in which case all DERs behave competitively. Finally, FIT is the Feed-In-Tariff scheme with the corresponding tariff be 0.099 as it currently is for wind installations in Greece. This value is also chosen as the maximum possible price that the coalition can set. The results are scaled in logarithmic scale.

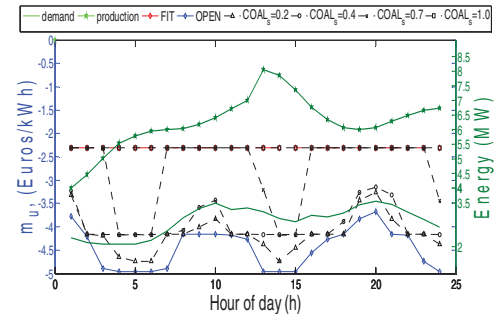


Fig. 3: Comparison of proposed algorithm (for different production shares) and FIT with respect to metric  $m_u$ . Green lines refer to the right axis.

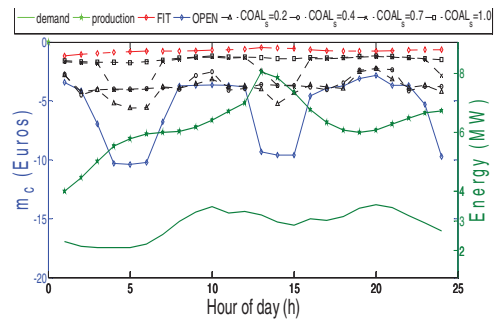


Fig. 4: Comparison of proposed algorithm (for different production shares) and FIT with respect to metric  $m_c$ . Green lines refer to the right axis.

Fig. 3 compares the performance of metric  $m_u$  with respect to the proposed algorithm (for various shares) and the FIT. It is obvious that the "OPEC" serves best the utility's perspective since it yields the lowest price. The worst performance for the utility is obtained for the FIT and the " $Coals=1.0$ ". While these two produce the same price, however, the "FIT" is much worse, since, apart from the high market price, it also forces the utility to buy the entire amount of energy produced. In contrast, the proposed algorithm results in buying the RES units at a price above than a completely liberalized market (" $Coals=0$ "), but less than the FIT policy depending on the share of the production units that the coalition has. More specifically, the higher the share of the coalition in the production, the higher the price of the RES units is. This means that the proposed strategy succeeds in making a smooth transition of current RES electricity market to more liberalized schemes. Fig. 4 depicts the performance of metric  $m_c$  with respect to the proposed algorithm and the FIT. In contrast to the previous metric ( $m_u$ ), the FIT policy is the best for the DERs, since it generates the highest profits. On the other side "OPEN" is the worst case and the proposed algorithm lies between the two extreme cases.

Again, it is obvious that the big coalitions benefit all participants in the RES market.

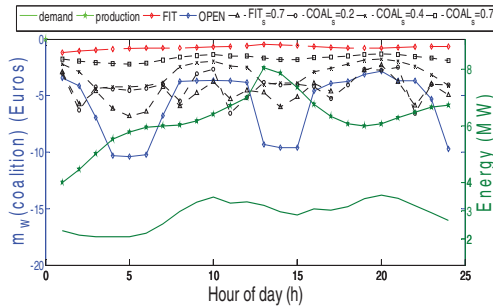


Fig. 5: Comparison of proposed algorithm (for different production shares) and FIT with respect metric  $m_w(\text{coalition})$ . Green lines refer to the right axis.

Figs. 5,6 show the performance of metric  $m_w$  for various examined sets comparing the proposed algorithm and the FIT. In particular, Fig. 5 examines only the DERs of the coalition. It is evident that the higher the production share of the coalition the better its profits, which means that the local DERs should prefer to create as big a coalition as possible. It is also noticeable that for small coalitions (see  $Coal_5=0.2$ , in fig.5) the resulting profits might be worse than the "OPEN" case. Therefore, there is a lower bound on the size of the association below which it is unsuccessful. The increase in profits of the proposed algorithm is coming due to a) the decrease in the number of competitors b) the closeness of the coalition to the target area and c) the increased market power that the coalition has. The union of DERs in one coalition implies that fewer competitors are involved in the market. As a result the competition is less intensive, which in turn results into a higher price in the market. This behavior is empowered from the coalition's market power, which chooses to slightly control the output in order to further hold the price at higher levels. Due to the closeness of the united DERs to the target area, the path losses are less resulting to larger margin for profits. Interestingly, we observe that if i) the coalition is big enough or ii) the demand is too high or iii) the target area is isolated from the main grid, the coalition's benefiting strategy might result to monopolistic tactics. This means that the DERs should be more benefited if they were united at peak periods or if a large coalition is built, since they can achieve unhindered higher prices. For the same reason, a regulator must ensure that the DERs outside the coalition can at least cover the requested demand, otherwise the electricity consumers will have to face an extremely unfair price. In this case, it is a good practice to set a high cap in the electricity price so that there is control in the coalition's behavior. In our simulations, we set the higher cap to be the FIT value for isolated areas (0.099 €/kWh). On the contrary, in case the rural area is connected with the main grid the above threat doesn't hold, since there is always enough supply from the grid to cover the entire demand. In this case, the proposed algorithm is an optimal solution for the DERs in the area to receive the maximum benefits from the decentralized market. In Fig. 6 we observe the distribution of profits among different categories of DERs for local coalition having the 70% of the total production. Each category is consisted with 20% of the total contained number of DERs in coalition. This can be derived from the metric  $m_w$  of the

examined sets. It is obvious that the algorithm favors the cheaper and the bigger DERs followed by very small DERs. It is rational that the most inefficient DERs are left with the least profits.

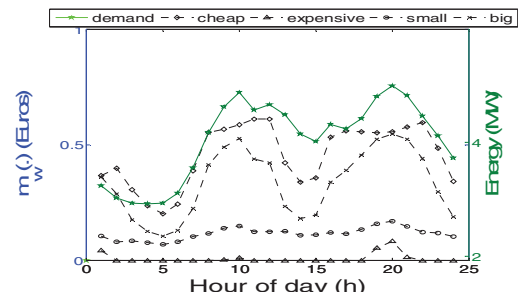


Fig. 6: Distribution of metric  $m_w$  of proposed algorithm among the different types of DERs in the coalition. Green lines refer to the right axis.

## VI. CONCLUSIONS

In this paper, we have proposed an algorithm that enables the local coalition to maximize its profits in a given geographical area under a decentralized market. Our simulations showed that there is an optimal price at which coalition's profits are maximized. We also observed that the bigger the coalition, the higher the profits for all the DERs in the market. This implies that a regulator should set an upper cap for the case where the coalition is too big. In general, if the rural area is connected with the main grid this threat doesn't hold, since the demand can always be covered by players outside the coalition. It is, also, evident that the proposed strategy succeeds in making a smooth transition of current RES electricity market to more liberalized schemes.

## ACKNOWLEDGMENT

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