

## Efficient Clustering of DERs in a Virtual Association for Profit Optimization

Vasileios Botsis  
Computer Technology  
Institute  
[botsis@ceid.upatras.gr](mailto:botsis@ceid.upatras.gr)

Nikolaos Doulamis  
National Technical  
University of Athens  
[ndoulam@cs.ntua.gr](mailto:ndoulam@cs.ntua.gr)

Anastasios Doulamis  
National Technical  
University of Athens  
[adoulam@cs.ntua.gr](mailto:adoulam@cs.ntua.gr)

Prodromos Makris  
Computer Technology  
Institute  
[pmakri@ceid.upatras.gr](mailto:pmakri@ceid.upatras.gr)

Emmanuel Varvarigos  
University of Patras &  
Computer Technology  
Institute  
[manos@ceid.upatras.gr](mailto:manos@ceid.upatras.gr)

**Abstract**—The Feed In Tariff policy (FIT) used for accelerating renewable energy investments cannot be retained as a sustainable business model for the future smart energy grid. It is also evident that the current centralized electricity market prevents small or very small energy producers, who usually generate energy by renewable means, to participate. In this paper, addressing the aforementioned problems, at first, we present a decentralized architecture (Virtual DER Clusters), where small Distributed Energy Sources (DERs) are united in coalitions, each participating in the market as a single entity. Then, efficient clustering algorithms are proposed based on a min-max optimization policy in order to dynamically derive the cluster that best satisfies coalition’s goals. Maximization in the sense of increasing as much as possible the profits of small-scale energy producers. Minimization in the sense of creating dynamically most competitive clusters. In this paper, three clustering policy schemes are discussed, each presenting different advantages with respect to the contradictory benefits between DERs and power utilities. From the examined policies, a fair sharing allocation scheme seems to be a good compensator between the electricity market and the small-scale players.

**Keywords**—min-max clustering, Fair sharing allocation, Virtualization, Virtual DER Clustering architecture

### I. INTRODUCTION

Feed-In-Tariff (FIT) policies have been recently used by many countries in order to stimulate rapid investments on Renewable Energy Sources (RES) [1]. FIT is effectively a subsidy policy, where small renewable energy generators are credited to sell their entire energy at higher prices than the normal market price. This, however, results in a total increase of the electricity price, especially when the policy “succeeds” and a large portion of energy is generated from DERs. This implies that we need to rethink the residual energy market model (and thus the respective supported Information and Communication Technologies-ICT) and replace FIT by a more competitive and market-oriented policy, without, however, preventing the further deployment of the renewable energy sector.

On the other side, direct participation of DERs to conventional liberalized electricity markets is not possible for many reasons. Firstly, conventional power plants sell their energy in a more competitive price than DERs since they exploit their mature technology. Secondly, RES producers are usually small-scale investments making difficult for them to directly participate in the electricity market. Therefore, we need new ICT tools able to allow the creation of renewable energy associations (clusters) through which very small energy producers can confederate together

to participate, in a more competitive way, into the electricity market. It should be noted that the move towards competitive markets means that the RES energy produced is more than the requested.

Toward this direction, recently, the European Union research area has funded the VIMSEN “Virtual Microgrids for Smart Energy Networks” project with the main purpose of creating ICT mechanisms that allow the creation of Virtual Microgrid (VMG) associations or clusters of DERs transforming the current centralized electricity market into a highly distributed one [2]. VMGs are associations of DERs networks that have agreed to operate together on a common basis. Each VMG is represented by an aggregator who is responsible for creating the most efficient cluster in order to achieve its goals (e.g., to produce a certain amount of energy, stemming from the requested demands, at the maximum possible benefit for the DERs associations). With this structure small-scale producers in electricity market are allowed to participate, since the VMG aggregator acts as a middleware between the power utilities and the small producers.

Recently, several research works have been proposed in the literature to cope with the role an aggregator in the electricity energy market. In [4], a two stage market model is introduced for power transactions. The proposed architecture includes an aggregator which acts as mediator between the producers and the demands of the power utility. The paper analyzes the pricing conditions for the aggregator and the utility so that they both gain for such a scheme. Moreover, if producers are not able to fulfill their obligations, they are penalized despite their intermittent nature. In [5] an interesting approach for demand shaping is presented, where the utility gradually increases monetary compensation to aggregators, who compete in offering their demand shaping units. Each aggregator compensates its customers for providing their units. In this scheme, it is derived that the most willing users do not derive the maximum benefits. In [6] a game theoretic framework is introduced to form coalitions that minimize distribution losses. More specifically, the main idea of [6] is to sequentially match each buyer’s demand while also minimizing the local transfer energy as much as possible

The main limitation of all the aforementioned approaches is that they handle the production of each producer independently one from the other. This means that no associations among them are permitted through the application of unsupervised clustering algorithms. Without a strong coalition among the DERs (which are usually small scale RES producers) their benefits are restrained, favoring more big electricity entities. Decentralized market schemes

implicate that the requested demand of an area will be covered locally. In this market, the local DERs, through an aggregator, can easily create a coalition and act as an individual. In [3] a decentralized architecture is presented, where a virtual aggregator (a software-based platform) allows DERs to trade electricity among them and the main market operator. In this work, the aggregation of DERs and the market operation are both performed by the aggregator and its purpose is to choose the cluster (the DERs) that will supply the market. The creation of cluster (for small-scale energy generators) from the aggregator of a decentralized market is the main contribution of this paper, however, in contrary to [3] we actively involve aggregator to the cluster creation in order to increase the revenues of the RES producers. . We also propose a new dynamic algorithm enabling fair distribution of market share among the DERs.

In particular, a min-max clustering scheme is adopted to generate the dynamic DERs associations. Maximization implies that selection of DERs in the clustering process is performed in a way to maximize their respective profits. Instead, minimization means that the created clusters are competitive as much as possible. Then, three main pricing policies schemes are discussed, each presenting different benefits for DERs, the aggregator and power utility. The first pricing policy favors power utilities which buy the demanded amount of energy at lowest possible cost. The second policy favors the most competitive DERs who sell their energy at maximum possible price. Finally, the third policy comprises the previous two by creating multiple clusters and fair-sharing the requested demand among them. The clusters are constructed in such a way that the requested energy demand is satisfied. This amount is set by regulation policies that urge power utilities (or other buyers) to acquire a portion of electricity energy by renewable resources in order to minimize environmental penalties. For example, an EU regulation obliges EU countries to include at least 20% from RES within their energy mix, or be subject to high penalties [7]. It should be noted that the move to more liberalized markets implies that the total output of the DERs is above the requested amount. This is an important assumption throughout this paper.

This paper is organized as follows: Section II describes the proposed decentralized market architecture. Section III discusses the distributed profit organization on a basis of independent DERs profiles. Cost production modeling is also described. The proposed virtual clustering concept is discussed in Section IV. The algorithms used to create the virtual clusters are analyzed in Section V. Experimental results are given in Section VI, while conclusions are drawn in Section VII.

## II. THE PROPOSED VIRTUAL DER CLUSTERING ARCHITECTURE

Virtual DER Clusters (VDC) are associations of small or very small scale producers (usually DERs) that have agreed to operate on a common basis (see Fig. 1). More specifically, each DER of VDC is connected through a gateway (GW) to an aggregator (VDCA), which is a software platform located to the web. The VDCA is the VDC middleware that

communicates with the utilities (market operator) or the other VDCAs in order to trade the RES production of its members. The VDCA has a database (DB) where it aggregates the information needed in order to group the DERs (builds a cluster) that best serve its purpose. The VDCA forms the cluster dynamically during different timeframes (eg. hourly interval or day-ahead), since energy demands and production temporally vary. Each small scale renewable energy producer announces to the VDCA the amount of energy that is forecasted to produce and the respective price. In case of an eminent agreement violation, eg. due to false forecast, VDCA can address other DERs and recreate the cluster dynamically in a closer time interval.

The future electricity market is expected to be decentralized, thus, VDCA communicates with the local market operator or another adjacent VDCA in order to define a certain amount of energy that should be offered by the VDC to a close target area. The requested energy amount is, therefore, location depended and VDCA is aware of the region that this requested demand is headed for. In the following, we denote as  $D(l)$  the demanded energy as requested from a region  $l$ . We have included dependency on region  $l$  since this affects the price that each DER offers its energy. Multiple demands (different locations) can also be requested from the same VDCA and, obviously, different clusters will be created. The satisfaction of multiple demands is out of the scope of this paper, nevertheless, the core of the cluster creation to satisfy a specific demand will remain.

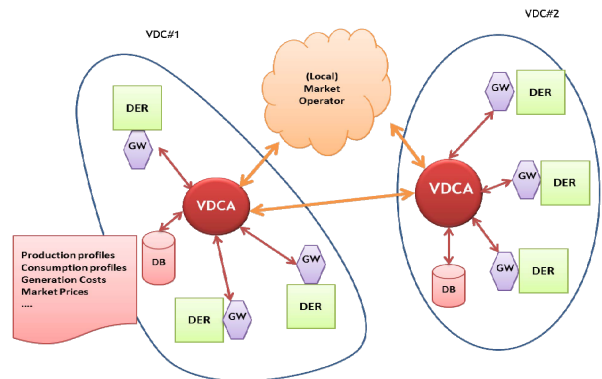


Fig. 1: Virtual DER Clustering Architecture

The GW has a central role in the VDC architecture. It is connected with smart metering interfaces in order to validate the energy amount offered by each small scale energy generator. Then, the VDCA decides the bid that it makes to the other players and the cluster that will provide the supply. It should be mentioned that the  $x_i$  refers to the *estimated* amount of energy that the  $i$ -th generator can produce, at a certain time interval, given by the metering information provided to the VDCA. After a successful agreement it instructs the cluster of DERs through appropriate actions of the GWs, implying that a telecom provider should develop communication infrastructure and protocols in order to forward the needed information to the aggregator. The work in [8] presents a scheme based on wireless sensor networks for enabling a reliable communication between the GWs.

### III. DISTRIBUTED PROFIT OPTIMIZATION OF INDEPENDENT SMALL SCALE PRODUCERS

#### A. Production Cost Modeling for Small Energy Producers

In order to formulate the clustering algorithms, we need to model the cost of producing  $x_i$  units of energy. Generally, the energy cost consists of two main factors; the energy production cost, say  $G_i(x)$ , and the transportation cost,  $T_i(x, l)$ . Function  $T_i(x, l)$  models cost of energy losses caused for delivering the  $x$  energy units from the physical location of the  $i$ -th DER to the region of demand  $l$ . Since resources of RES production are free the marginal cost of producing a unit of energy is limited to the fixed operational and maintenance costs that the investment needs. However, small scale producers are additionally consumers. As a result, they have two choices for the produced energy: either to sell it to the system or consume it onsite. Obviously, the first option should normally force them to buy the corresponding amount of energy from the power company at a specific price. While this, in generally, is not reasonable due to the additional transportation losses, nevertheless, if RES units are in great demand (eg. from an industrial plant to avoid emission penalties), then the selling of the RES units in the system implies higher profits than consume them onsite. Therefore we assume that the generation cost of RES units is the cost of buying the corresponding amount of energy from the power company, which is usually linearly dependent:

$$G_i(x) = \begin{cases} \gamma_i \cdot x, & 0 \leq x \leq x_{i,max} \\ \infty, & \text{otherwise} \end{cases} \quad (1)$$

where parameter  $\gamma_i$  models the price of electricity for buying  $x$  energy units from the power company. Variable  $x_{i,max}$  refers to the maximum generated energy. Normally, fixed costs should also be present in order to represent costs like the connection to the main grid from the DSO, or for the services that the aggregator charges the DERs. However, throughout this paper we ignore these costs since they don't influence the general clustering algorithms.

Transportation costs,  $T_i(x, l)$  can be decomposed into (i) the power losses over the line, and (ii) the cost of transforming the energy from low voltage to high voltage and vice versa. Then, the transportation costs for the  $i$ -th DER to distribute the  $x$  units to region  $l$  are: That is,

$$T_i(x, l) = \gamma_i \cdot (R_i(l) / V_0^2 \cdot x^2 + \delta_i \cdot x) \quad (2)$$

where  $\delta_i \cdot x$  models the transformation cost and  $R_i(l) / V_0^2 \cdot x^2$  models the power losses over powerline with resistance  $R_i(l)$  and voltage  $V_0$  respectively[9].  $\gamma_i$  is the price of electricity as indicated in (1).

Equation (2) implies that the cost of the  $i$ -th MG is dependent on its distance from the target region. Note that, since we must have:  $(1 + \delta_i) \cdot x + R_0(l) / V_0^2 \cdot x^2 < x_{i,max}$ , we get for the maximum quantity that can be offered in region  $l$ :

$$x_{i,max}(l) = V_0^2 \cdot \frac{-1 - \delta_i + \sqrt{(1 + \delta_i)^2 + 4 \cdot \frac{R_i(l)}{V_0^2} \cdot x_{i,max}}}{2 \cdot R_i(l)} \quad (3)$$

This means that the maximum supply a DER can deliver, varies depending on the region. Unifying the above costs we obtain that the total cost of  $i$ -th producer for supplying  $x$  units to region  $l$  is of quadratic form:

$$C_i(x, l) = G_i(x) + T_i(x, l) = \begin{cases} a_i x + b_i(l)x^2, & 0 \leq x \leq x_{i,max}(l) \\ \infty, & \text{otherwise} \end{cases} \quad (4)$$

where  $a_i = \gamma_i \cdot (1 + \delta_i)$ ,  $b_i(l) = \gamma_i \cdot R_i(l) / V_0^2$ .

We should mention that in this paper we make the assumption that there are no technical constraints in the transmission system (eg. capacity of powerlines), which in fact are the main challenge in transmitting energy generated from DERs.

#### B. Profit Optimization

Each DER in the market tries to maximize its profit  $\Pi_i$  by solving eq.(5):

$$\max_{\lambda} \{ \Pi_i = \lambda_i \cdot x_i - C_i(x_i, l) \} \quad (5)$$

where  $\lambda_i \cdot x_i$  denotes the revenue of the  $i$ -th DER for price  $\lambda_i$  and production  $x_i$ .

Given the price  $\lambda_i$ , the solution of eq.(5) with respect to  $x_i$  gives the optimal amount of units  $x_i$  that the  $i$ -th DER must offer to region  $l$  in order to maximize its profits. In contrary, given the amount  $x_i$ , the solution of eq.(5) gives the optimal price for  $i$ -th DER at which its profits are maximized [see eq.(6)]. In particular,

$$\hat{\lambda}_i(x_i, l) = C'_i(x_i, l) \quad (6)$$

where  $C'_i(x_i, l)$  represents the derivative of the eq.(4).

Since the cost function is monotonically increased, initially the maximum profit for the  $i$ -th DER is provided by its maximum capacity, i.e.,  $x_i = x_{i,max}(l)$ . This is due to the fact that renewable energy cannot be stored (apart perhaps for a small amount into batteries) and/or there is no fuel consumption that can increase resources savings. The  $i$ -th DER then announces to the aggregator its optimal price and the respective production capacity to offer to the system. This is done independently for each DER to maximize its own profit. Note that both the delivered units  $x_i$  and the price  $\lambda_i$  depend on the location  $l$ , due to the transportation costs. This means that DERs placed on the regions nearby to the demand will present lower prices against far away DERs. The aggregator knows the target area of the energy demand and, therefore, each DER announces to the aggregator a pair of each optimal value for region  $l$ :

$$(x_i^*, \lambda_i^*, l) \text{ with } x_i^* = x_{i,max}(l) \text{ and } \lambda_i^* = \lambda_i(x_{i,max}(l)) \quad (7)$$

In the following, we omit variable  $l$  for simplicity purposes since we refer to a specific location.

#### IV. VIRTUAL DER CLUSTERS

Let us assume that a set  $S$  is created from an association of several DERs. All DERs of the same cluster share a common agreed price to the market, say,  $\lambda(S)$ . In this case, variable  $\lambda(S)$  refers to the common price of the association and not to the price of an individualized DER.

##### A. Energy Demand Underflow

Let us now consider the case that the maximum total capacity of all DERs of the cluster is strictly less than the requested energy demand  $D$ . Therefore,  $\sum_{i \in S} x_{i,max} = X(S) < D$  (energy demand underflow), where  $X(S)$  denotes the capacity offered by all members of the cluster. It is clear that in this case, all DERs will operate at their maximum capacity  $x_{i,max}$  as in Eq.(7). Thus, the aggregator estimates an optimal agreed (common) price  $\lambda^*(S)$  that maximizes the profit among all members of the association  $S$ :

$$\begin{aligned} \lambda^*(S) &= \arg \max_{\lambda} \left\{ \lambda(S) \cdot \sum_{i \in S} x_i - \sum_{i \in S} C_i(x_i) \right\} \\ \text{s.t. } \sum_{i \in S} x_i &= X(S), 0 \leq x_i \leq x_{i,max} \end{aligned} \quad (8)$$

Clearly, at least one DER should be added in the cluster in order to meet requested demand  $D$ . Let us now assume that the aggregator selects the  $j$ -th DER of maximum capacity  $x_{j,max}$  to be added to the cluster. Then, this DER will announce to the aggregator, apart from its maximum capacity, its most beneficial price  $\lambda_j^*$  derived from Eq.(7) when the  $j$ -th DER operates at its maximum capacity, i.e.,  $x_j = x_{j,max}$ . If again,  $x_{j,max} + X(S) \leq D$ , then optimization of Eq.(7) results to the fact that the new common price for all members of the augmented cluster  $S' = S \cup \{j\}$  should be the maximum between the two prices:

$$\lambda^*(S') = \max\{\lambda^*(S), \lambda_j^*\} \quad (9)$$

Equation (9) means that if the aggregator assembles into the association some DERs that can produce energy at a lower (cheaper) price than the common price of the cluster, the competitiveness of the cluster will increase. This is due to the fact that the new cluster will offer more total capacity but at the same cluster price, as from Eq.(9). On the other hand, if there is no a cheaper DER available, then the optimal choice is the select the DER that offers the lowest price. This way, the new common price of the association will be increased as minimum as possible. In case that several DERs of the same price are available, the optimal choice is to select the one of the highest energy production units. This lead to the following Statement:

*Statement 1: The cheapest DERs should be preferred from an association of DERs that produces energy demand underflow. In case that several DERs are available with the same price, the one of the maximum capacity are preferable.*

Note that if total supply is less than the requested demand, then, the above generation cost function limits the price to the price of buying  $X(S)$  units of electricity in retail market. This implies that a price cap needs to be set by a

regulator in case that the cost functions are hidden from the aggregator.

##### B. Energy Demand Overflow

Let us now examine the case where  $x_{j,max} + X(S) > D$ . Then, at least one DER belonging to the new augmented  $S'$  association should offer a lower energy capacity than  $x_{i,max}$ ,  $i \in S'$ . This required amount is determined by the aggregator which also adjusts the price given to DERs. The new optimal capacity for each DER is estimated by solving the following convex constrained optimization system.

$$\begin{aligned} \max_{x_i, x_j} & \left\{ \lambda(S') \cdot \left( \sum_{i \in S} x_i + x_j \right) - \sum_{i \in S} C_i(x_i) - C_j(x_j) \right\}, \\ \text{s.t. } \sum_{i \in S} x_i + x_j &= D, 0 \leq x_i \leq x_{i,max}, 0 \leq x_j \leq x_{j,max} \end{aligned} \quad (10)$$

The entrance of a new DER in the cluster forces the previously allocated DERs to reduce their total offered amount of energy in order to maintain the offered supply equal to  $D$ . Since the DERs of  $S'$  offer lower supply than previously, this leads also to lower price and the next statement:

*Statement 2: If a cluster has total capacity that can satisfy demand, then the price obtain for the same demand by adding a new MG in the cluster, is equal or less to the previous price.*

Relaxing the capacity constraints ( $0 \leq x_i \leq x_{i,max}$ ) the aforementioned equation can be solved analytically. However, in practice, the most convenient way is the application of a distributed optimization process. In this case, the aggregator iteratively reduces the common price of the association in order to satisfy the constraint  $\sum_{i \in S} x_i + x_j = \sum_{i \in S'} x_i = D$ .

$$\lambda(k+1, S') = \lambda(k, S') + \varphi \cdot [D - \sum_{i \in S'} x_i] \quad (11)$$

where  $k$  is the iteration index and  $\varphi$  a constant variable which denotes the reduction step of the algorithm.

Algorithm 1 summarizes the main steps of the iterative algorithm used to find the optimal common price for a cluster  $S$ . In particular, the algorithm initially selects as the common agreed cluster price a random price of an individualized DER of the cluster as defined by eq.(7) for which supplies of each DER are derived using eq.(6). Then, the common price is updated according to eq.(11) in case that the constraint is to not satisfied, that is  $\sum_{i \in S'} x_i \neq D$ .

Eq.(11) reduces the common price at the next iteration of the algorithm when the total offered capacity exceeds the requested one. The opposite happens when the offered capacity is less than the requested demand. The algorithm stops until equilibrium is reached that yields the optimal price at the requested energy  $D$ . Due to the fact that the algorithm matches the requested demand we call it "Demand Fit" algorithm (DFA).

---

##### Algorithm 1: The Demand Fit Algorithm

---

1. Set as initial common optimum price a random price of an

---

- 
- individualized DER of the cluster. Set  $k:=0$ .
2. Estimate the optimal production units  $x_i$  for all DERs of the cluster using the initial price  $\lambda(k=0)$ . This is performed by maximizing the respective profit of each DER by solving eq.(6) with respect to  $x_i$ .
  3. While (total capacity is different than  $D$ )
    - 3.1. Set  $k:=k+1$
    - 3.2. Update cluster price  $\lambda(k)$  as in eq.(11)
    - 3.3. For all DERs in the cluster do
      - Update DER production units by solving eq.(6) with respect to  $x_i$
  4.  $\lambda(k)$  is the current common optimum cluster price
- 

From the previous Statements it is clear that if the aggregator optimizes the benefits of power utilities, then the optimal approach is to create one cluster that includes all DER.

### C. Virtual DER Clustering Algorithm

In this section, we describe how the clusters are created within the association. Let us initially assume that we have  $N$  available DERs, each announces to the aggregator pair  $(x_i^*, \lambda_i^*)$  of their produced energy units and the respective price. Then, exploiting the concepts of Statement 2, it is clear that if the aggregator selects the smallest possible number of DERs that can satisfy the demand, then this amount yields the highest possible price for this cluster, since adding more members in it would reduce the price for all of them. Consequently, in order to maximize cluster's benefits, the cluster should be the one formed by the cheapest DERs, but including the smaller possible number of DERs able to satisfy the demanded amount.

This is in fact a *min-max policy*. Maximization means that we select the DERs in a way to maximize their profits (the smallest adequate number within a cluster able to satisfy the requested demand). Minimization means that among all potential clusters, we select the one that yields the lowest (most competitive) price.

Taking into account the aforementioned arguments, the algorithm for constructing  $K$  clusters is described in the following Algorithm 2. Since the clusters follow a min-max policy we call it the "K-Min-Max Algorithm" (K-MMA). It should be clear that price offered by  $j$ -th created cluster is lower than price offered by  $j+1$ -th created cluster (i.e.  $\lambda^*(S_j) \leq \lambda^*(S_{j+1})$ ).

---

#### Algorithm 2: The K-Min-Max Algorithm

---

1. Collect all the pairs  $(x_i^*, \lambda_i^*)$  for all  $N$  DERs
  2. Sort the DERS with respect to  $\lambda_i^*$  in an ascending order.
    - For those DERs that share the same price, sort them with respect to their production units in a descending order.
  3. Create a list that contains the sorted DERs, *Sort\_DER*[ $N$ ]
  4. Set  $k:=1$  and *Index*:=1
  5. Set the total energy capacity of the cluster  $X(k):=0$ .
  6. While  $X(k)<D$  or *Index* $\leq N$  do:
    - 6.1. Expand the cluster:  $S_k = S_k \cup \{i\}$ , where  $i:=\text{Sort\_DER}[\text{Index}]$ .
- 

- 
- 6.2. Update  $X(k):=X(k)+x_{i,\max}$
  - 6.3. Update *Index*:=*Index*+1
  7. Set  $k:=k+1$  go to Step 5.
- 

## V. VDC CLUSTERING POLICIES

### A. The "AllTogether" Policy

From the previous Statements it is clear that if the aggregator optimizes the benefits of power utilities, then the optimal approach is to create one cluster that includes all DER in the association. This is the only case when the total capacity offered is less than the requested one, i.e., the production of all available DERs cannot satisfy the electricity demands. Then, only a lower demand can be satisfied and this will be achieved at the maximum possible price grouping all DERs into one cluster. In other words, all the DERs will gain by such a scheme since the requested energy amount exceeds the produced one.

In case that the total capacity of all DERs is greater than the requested demand, the agreed price should be smaller than any other case [see Statement 2] making the association profitable for the power utility (it will buy the energy at the lowest possible cost). This is valid due to the DFA [see Algorithm 1].

### B. The "OneCluster" Policy

In this policy, the cheapest DERs that can offer the demanded energy at the lowest price will be selected as the cluster to supply the market. Using the K-MMA, this cluster is the  $S_j$ . It should be mentioned that Clusters  $S_j$  have been created in a way to *maximize the profit of the participant DERs*. However, such a selection has a risk. It is possible all the remaining DERs to confederate together and offer the demanded energy amount at a lower price than the cheapest cluster. This is achieved since, from Statement 2, as we increase the number of members in a cluster that can satisfy the requested energy, then price within that cluster will gradually decrease (probably to a price lower than offered by  $S_j$ ). More specifically, if we denote as  $A$  the set of the cheapest DERs cluster and  $A'$  the set of the remaining ones (complementary cluster) and as  $\lambda(A)$  and  $\lambda(A')$  the respective cluster price, then the aggregator's goal of maximizing profits of cheapest DERs is written as:

$$\max_{\lambda} \{ \lambda(A) \} \leq \lambda(A') \quad (12)$$

where  $\lambda(A)$  and  $\lambda(A')$  are obtained by applying the DFA in clusters  $A$  and  $A'$  respectively for acquiring  $D$  units.

This case is particularly evident when the total offered energy is much greater than  $D$ . In that case, it is quite common the overwhelming majority of the DERs to be excluded from the most competitive cluster. The total capacity of these excluded DERs is anticipated to be times greater than  $D$  and thus if they confederate together they can offer a lower price than the most competitive cluster ( $S_j$ ) to win the competition stage. Instead, if the total capacity possessed by all DERs is somehow greater than the requested energy  $D(l)$  then it is quite improbable the remaining DERs to manage to offer the requested amount forcing the winner towards the most competitive cluster side.

### C. The “FairSharing” Policy

To handle the aforementioned problem, a new fair sharing strategy is also examined in this paper. This strategy is valid in case that the total offered capacity from all DERs is greater than the requested one. In particular, we exploit the contribution of all DERs to estimate the common optimal price instead of using only the most competitive DERs. This way, we do not exclude some DERs from the association which may cause the termination of their cooperation with the aggregator. The difference of this policy with respect to the one presented in Section V.A, is that the clusters have been created in a way to maximize their profits and therefore they declare to the aggregator the maximum possible price that they can receive in order to satisfy the demand [see K-MMA]. On the contrary, the policy of Section V.A reduces the offered price to equals to the most competitive one.

Let us first assume that  $\sum_{all\ i} x_{i,max} = n \cdot D + e$ , where  $e$  denote the energy capacity offered by the last created cluster. It is clear that  $e \leq D$ . Variable  $n$  refers to the number of times that the total offered energy capacity exceeds  $D$ . Let us also denote as  $K$  the number of clusters created by K-MMA and  $S_j$ ,  $j=1,2,\dots,K$  the respective clusters. It is clear that the capacity of the  $K-1$  clusters equals  $D$ , i.e.,  $X(S_j)=D$ ,  $j=1,2,\dots,K-1$ , while  $X(S_K)=e \leq D$ . We also denote as  $\lambda^*(S_j)$  the optimal price for the cluster  $S_j$ . Recall that  $\lambda^*(S_1) \leq \lambda^*(S_2) \leq \dots \leq \lambda^*(S_K)$ . Then, a fair sharing algorithm is adopted to adjust the exceeded energy capacity in order to satisfy the demand  $D$ .

#### 1) The Proposed Fair Sharing Algorithm

Initially, we form a set of weights that are assigned to the  $K$  created clusters.

$$w_j = \begin{cases} 1 - D \cdot N_j \cdot \lambda^*(S_j) / \sum_{all\ j} \lambda^*(S_j), & j = 1, \dots, K-1 \\ 1 - e \cdot N_j \cdot \lambda^*(S_j) / \sum_{all\ j} \lambda^*(S_j), & j = K \end{cases} \quad (13)$$

where  $N_j$  the number of DERs in the  $j$ -th cluster.

The weights of Eq.(13) assign a higher value for the most competitive clusters and a smaller value for the other ones. Then, these weights are taken into account in the energy allocation process. More specifically, the higher the weight value is, the higher the allocated units to this cluster, which in turn yields higher profits. As a result, the MGs will prefer to participate in the cheapest clusters, otherwise less demand will be assigned to them. On the other hand, the expensive MGs (which are usually very small scale producers) are not excluded from the market thus, incentivizing their investments. Further demand allocation is also needed within each cluster among the composing DERs. In both cases, the demand allocation is done using the weighted max-min fair share algorithm that allocates a given resource among users based on weights 0.

In order to understand the fair sharing optimization policy, let us concentrate in the following on a simple example. Let us, first assume that we have  $D=9.6$  units to allocate among four clusters. The cluster capacities are [8, 1, 1, 14] and the corresponding weights are [2, 1, 1, 2]. These

weights have been obtained from the optimal prices per cluster. In this example, we have scaled the weights of eq.(13) to take integer values instead of laying in the interval [0,1] and the higher the weight, the higher the priority. Our goal is to allocate the  $D$  units among the four clusters in a fair way. Initially, the algorithm divides the  $D$  units into  $2+1+1+2=6$  equal parts, that is,  $9.6/6=1.6$  units. Then, clusters 1 and 4 receive  $2*1.6=3.2$  units, while clusters 2 and 3 receive  $1*1.6=1.6$  units. However, the offered capacity of clusters 2 and 3 is 1, leaving a residual of  $0.6*2=1.2$  units unused. In the next round the unused 1.2 units are again fairly shared among the two clusters that didn't fulfill their capacity. This is done by dividing the 1.2 units into  $2+0+0+2=4$  equal parts, that is,  $1.2/4=0.3$ . Then, clusters 1 and 4 receive an additional  $2*0.3=0.6$  units. Since the total residual at this step is zero, the algorithm stops. The final energy capacity allocation among the four clusters is the [3.8, 1, 1, 3.8] that equals the demand of 9.6 units.

The fair sharing algorithm defines the total amount of energy that a cluster sells to the aggregator. This optimal amount of total energy production per cluster is denoted as  $X^*(S_j)$  for the cluster  $S_j$ .

The next step is to define the percentage of reduction for all DERs that participate in the cluster in order to satisfy the total production capacity  $X^*(S_j)$ . This again is performed by the application of the fair-sharing optimization scheme but now within cluster. DERs within a cluster  $S_j$  have weights that are obtained from their supply for  $D$  units (or  $e$  for the  $K$ -th cluster):

$$w_{i,DER} = x_i / D, \quad \forall i \in S_j \quad (14)$$

The optimal agreed price per cluster  $\lambda^*(S_j)$  is readjusted for  $X^*(S_j)$  using the DFA. Note that the market price  $\lambda^*$  that the aggregator offers its units to the local market is simply the average price that the involved DERs offer their units. That is:

$$\lambda^*(\cup S_j) = \left( \sum_{all\ j} \lambda^*(S_j) \cdot X(S_j) \right) / \sum_{all\ j} X(S_j) \quad (15)$$

## VI. SIMULATION RESULTS

### A. Metrics

We used three metrics to assess the gains of the proposed clustering models:

$$m_u = \lambda^* \quad (16)$$

$$m_c = \sum_{i=1}^M \lambda_i^* \cdot x_i - \sum_{i=1}^M C_i(x_i) \quad (17)$$

$$m_w = \frac{1}{|S_w|} \cdot \left( \sum_{i \in S_w} \lambda_i^* \cdot x_i - \sum_{i \in S_w} C_i(x_i) \right) \quad (18)$$

where  $\lambda^*$  comes from the eq.(11), (12) or (15) depending on the applied algorithm.  $M$  stands for the number of DERs in the market,  $S_w$  is the set of the DERs that win the market and  $\lambda_i^*$  the price of cluster that those DERs (the winners) sell



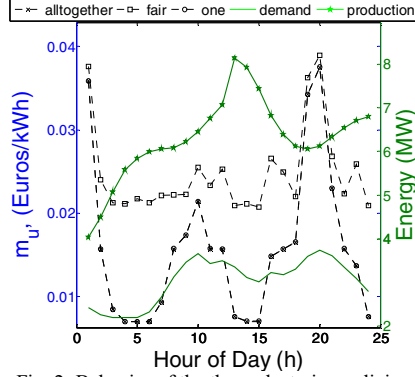


Fig. 2: Behavior of the three clustering policies with respect to metric  $m_u$ . The lower the value the better for the power utilities. Bold lines refer to the right axis.

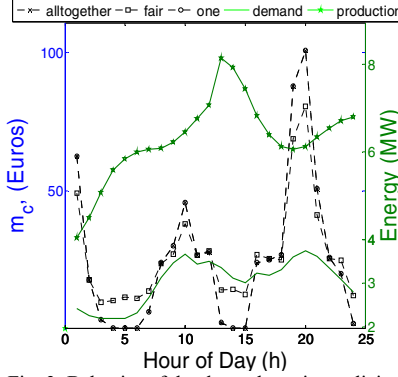


Fig. 3: Behavior of the three clustering policies with respect to metric  $m_c$ . The higher the value, the better for the DERs of the market. Bold lines refer to the right axis.

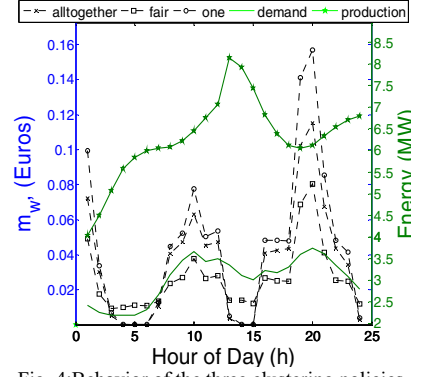


Fig. 4: Behavior of the three clustering policies with respect to metric  $m_w$ . The higher the value, the better for the winning DERs of the market. Bold lines refer to the right axis.

their units (i.e. if  $i \in S_j$ , then  $\lambda_i^* = \lambda^*(S_j)$ ).  $x_i$  is the supply of the  $i$ -th DER.

**Metric  $m_u$ :** This metric represents the price at which power utility buys the requested amount  $D$ . Obviously, the lesser the price the bigger the benefit for power utility.

**Metric  $m_c$ :** This metric defines the aggregated profits of all DERs in the market. Obviously, for DERs, the higher the value of  $m_c$ , the better for them.

**Metric  $m_w$ :** This metric derives the average profits of the DERs that manage to supply the market (i.e. the winners). The higher the average profits the better for the winning DERs.

### B. Simulations

For our simulations, we set up a distribution network within a square of  $150 \times 150 \text{ km}^2$  with the target area (the point which the demand is headed for) and the DERs randomly deployed within this area. The resistance between any two nodes (DERs) and the voltage line are set to  $R_0 = 0.2$  ohm per km and to 230V, respectively, which are practical values in the lower level of the distribution networks [9]. For simplicity purposes, we assumed that  $\delta_i = 0$ . Typical small scale installations have capacities within the range [1kW-30kW], therefore, the capacities of DERs are assumed to be Gaussian random variable with mean 15kW. In order to evaluate realistic experiments of a day-ahead market, statistical data from Sardinia's and Greek electricity market were used. In particular, production output of wind installations, as well as electricity demand within Sardinia on 23/02/2015 were used for the definition of hourly production pattern of the DERs and the corresponding hourly demand [11]. For the definition of  $\gamma_i$  values we used realistic consumption patterns of households using dataset of [12]. Then, the  $\gamma_i$  parameters were matched to the corresponding electricity prices of [13]. Finally, due to computational limitations we used 1000 DERs able to cover a smaller portion of the actual demand. Nevertheless, without any loss of the validity of our results we used the above daily demand pattern in the range [2-4]MW. All statistical results are averaged over a large number of runs with different random positions, capacities and cost parameters for the DERs.

### C. Results

In Figs.2-4 we depict the performance of the algorithms with respect to the metrics defined in previous subsection. In the above figures the “alltogether”, “fair”, “one” stand for the “alltogether” policy [section V.A], the “fair sharing” policy [section V.C] and “one cluster” policy [section V.B] respectively. The demand is the hourly overall requested demand from the RES producers, while the “production” is the overall production of the DERs.

Fig. 2 presents the performance of the metric  $m_u$  with respect to different clustering policies. It is obvious that the “alltogether” strategy serves best the utility's perspective, since it yields the lowest price. Very close to the “alltogether” follows the “one” policy, who gives slightly increased prices. On the other hand, the “fair-sharing” policy yields the highest price and, thus, is less beneficial for utilities. In all cases, we can easily see that the price curve is affected by the portion of the total production divided by the demand. It is also noticeable that the “fair” is the most dependent on the above portion, since the more the capacity of the coalition in terms with the demand, the higher the resulted price is. This implies that the DERs should prefer to create large coalitions with large capacities.

Fig. 3 presents the performance of the clustering policies with respect to metric  $m_c$  (total profits). We observe that the “fair”, in general, yields the higher total profits making it more suitable from the DERs' perspective. In consistence with the previous results for metric  $m_c$ , “alltogether” is the worst strategy for DERs, with the “one” being slightly better (than “alltogether”). It is also noticeable that in peak hours (e.g. 20:00) the “fair” has worse performance than the others. This is due to the fact that the “fair” grabs a portion from the cheaper DERs' revenues and allocates it to more expensive. As a result total profits are reduced. Nevertheless, if the total supply in coalition is many times larger than the demand every DER is benefited in the “fair” scheme”. Fig. 4 depicts the impact of the tree strategies over the average profits of the winning cluster. It is obvious that the “one” strategy is the most beneficial for the winners, since those DERs receive the highest average profit. However, “one” is also the most harmful for the inefficient investments (usually very small

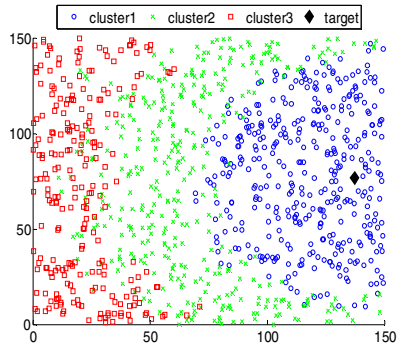


Fig. 5: Performance of K-MMA in a market with small deviation in productions of DERs. “Target” is the location of the area that the demand is headed for.

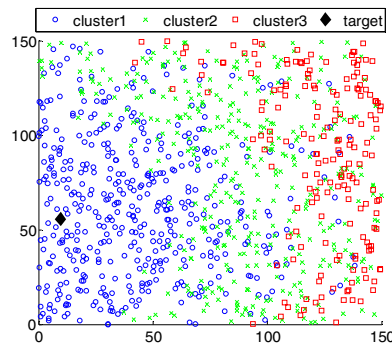


Fig. 6: Performance of K-MMA in a market with medium deviation in productions of DERs. “Target” is the location of the area that the demand is headed for.

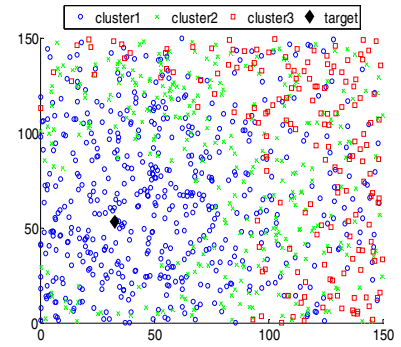


Fig. 7: Performance of K-MMA in a market with large deviation in productions of DERs. “Target” is the location of the area that the demand is headed for.

scale) since it lets those DERs (that don’t win the market) to starve. In contrast, “fair” distributes the profits among all the DERs of the coalition entailing twofold benefits. Firstly, the cheapest DERs receive a higher revenue stream than the other strategies, especially in case that the requested demand is too small. Secondly, it should be evident that, in “fair”, none DER is allowed to have zero profits. In other words, even the more expensive DERs generate profits. Finally, we can see in Fig. 4 that the “fair” provides a steadier income than the others strategies removing financial uncertainties for the DERs. Therefore, this policy gives incentives to DERs in order to be competitive without deterring further small-scale RES investments.

In Figs.5-7 we present the performance of K-MMA for DERs having i) very small deviation (Fig. 5) ii) medium deviation (Fig. 6) and iii) large deviation (Fig. 7) in production capacities. In these experiments we also assumed that the DERs deployed in the area have the same  $a_i$  parameters. “Target” is the location of the area that the demand is headed for. We can see that in the first case the clusters are created in a location-based manner. The closer DERs constitute the first (and more important) cluster, the second constitute the next closer DERs e.t.c. It should be noted that, in general, the transportation costs ( $b_i$  parameters) are not necessarily connected with the DER’s distance from target area (i.e. a DER in a close island might have greater transportation cost than a DER in a more distant mainland). In contrast, if the production of DERs has larger deviation (as in the next two cases), then we observe that the clusters are created in a more location-distributedly manner (Fig. 5, Fig. 6). Obviously, the larger the deviation the more distributed the clustering is. This behavior comes due to the fact that, individually, the greater the injection in the transmission system, the greater the transportation costs (since cost function is convex). In this case, costs are reduced if DERs located farther (from target area) can offer a small portion of their supply. Of course, this makes difficult for the system operator to implement the power transactions, which is the main limitation of distributed generation.

## VII. CONCLUSIONS

The VDC architecture enables the small-scale DERs to participate in the electricity market far from FIT towards more decentralized and liberalized market schemes. The move to competitive market schemes implies that the total production of DERs is more than the requested amount. Using this assumption, in this paper, we introduce a new dynamic algorithm that allows the creation of efficient DER clusters in a decentralized market. Then, three policies are examined through the use of a min-max scheme to maximize DER benefit while simultaneously keeping the cost as low as possible. A fair sharing allocation scheme seems to be a good compensator between the electricity market and the small-scale players, since it benefits more the efficient DERs, whilst not allowing any DER to have zero profits.

## ACKNOWLEDGMENT

This work is supported by VIMSEN “Virtual Microgrids for Smart Energy Networks”, EU grant: FP7 ICT 619547.

## REFERENCES

- [1] I. Couture et al. "An analysis of feed-in tariff remuneration models: Implications for renewable energy investment." Energy Policy, 2010.
- [2] VIMSEN, "Virtual Microgrids for Smart Energy Networks," European Research Project, [on line available] <http://www.ict-vimsen.eu/>.
- [3] S.You, C.Træholt, and B.Poulsen, "A market-based virtual power plant," IEEE Int. Conf. on. Clean Electrical Power, 2009.
- [4] H. Kim, and M. Thottan, "A two-stage market model for microgrid power transactions via aggregators," Bell Labs Techn. Journal, 2011.
- [5] L. Gkatzikis et al., "The role of aggregators in smart grid demand response markets," IEEE Journal on Selected Areas in Comm., 2013.
- [6] W. Saad, Z. Han, H. V. Poor, "Coalitional game theory for cooperative micro-grid distribution networks," ICC, 2011.
- [7] EU Directive 2009/28/EC of the EU Parliament, 23 April 2009
- [8] H. Li and W. Zhang, "QoS routing in smart grid," GLOBECOM, 2010
- [9] J. D. Glover, M. Sarma, and T. Overbye, "Power System Analysis & Design", Cengage Learning, ISBN-13: 978-1111425777, 2011.
- [10] N. Doulamis et al., "Fair scheduling algorithms in grids", IEEE Trans. on Parallel and Distributed Systems, Vol. 18(11), 2007.
- [11] <https://transparency.entsoe.eu/>
- [12] I.Richardson, et al., "Domestic electricity use: A high-resolution energy demand model", Energy and Buildings, 42(10), 2010.
- [13] <https://www.dei.gr/Documents2/TIMOLOGIA/TIMOL25072014/OIKG1NMEXRONO.pdf>